

## UJIAN AKHIR SEMESTER MATEMATIKA I

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OPEN BOOK

150 MENIT

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1) Tentukan  $dy/dx$  jika diketahui persamaan dibawah ini.

a)  $y = \left(\frac{4}{x^2} + 1\right)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{4}{x^2} + 1\right)^{-\frac{1}{2}} \left(\frac{8}{x^3}\right) = -\frac{4}{x^3 \left(\frac{4}{x^2} + 1\right)^{1/2}} = -\frac{4}{x^3 \sqrt{\frac{4+x^2}{x^2}}} = -\frac{4}{x^3 \sqrt{4+x^2}}$$

b)  $y = \ln \sqrt{\frac{x+2}{x-2}}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{\frac{x+2}{x-2}}} \times \frac{1}{2} \times \frac{1}{\sqrt{\frac{x+2}{x-2}}} \times \left\{ \frac{(x-2) - (x+2)}{(x-2)^2} \right\} \\ &= -\frac{2}{\left(\frac{x+2}{x-2}\right)(x-2)^2} = -\frac{2}{(x+2)(x-2)} = -\frac{2}{(x^2-4)} \end{aligned}$$

c)  $xy + y^2 - x^2 = 5$

$$\frac{d}{dx}(xy) + \frac{d}{dx}(y^2) - \frac{d}{dx}(x^2) = \frac{d}{dx}(5)$$

$$y + x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) - 2x = 0$$

$$y - 2x + (x + 2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x - y}{x + 2y}$$

2) Tentukan  $df(x)/dx$  dari fungsi-fungsi di bawah ini.

a)  $f(x) = \cos^3(2x-3)^2$

$$\begin{aligned} \frac{df}{dx} &= [3\cos^2(2x-3)] [-\sin(2x-3)] [2(2x-3)2] \\ &= -12(2x-3)\cos^2(2x-3)\sin(2x-3)^2 \\ &= (36-24x)\cos^2(2x-3)\sin(2x-3)^2 \end{aligned}$$

b)  $f(x) = \tan x^2 + \tan^2 x$

$$\frac{df}{dx} = 2x \sec^2 x^2 + 2 \tan x \times \sec^2 x$$

c)  $f(x) = \frac{x}{\sin x + \cos x}$

$$\frac{df}{dx} = \frac{(\sin x + \cos x) - x(\cos x - \sin x)}{(\sin x + \cos x)^2} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}$$

3) Diketahui sebuah fungsi:

$$f(x) = \frac{x^2 - 1}{x^2 - 3}$$

a) Domain fungsi.

$$f(x) = \frac{x^2 - 1}{x^2 - 3} = \frac{(x-1)(x+1)}{(x-\sqrt{3})(x+\sqrt{3})}$$

$$f(x) = \{x \mid x \in R, x \neq \sqrt{3}, x \neq -\sqrt{3}\}$$

b) Titik potong kurva  $f(x)$  dengan sumbu koordinat.

$$f(x) = 0 \quad \text{jika } x = 1 \Rightarrow (1, 0)$$

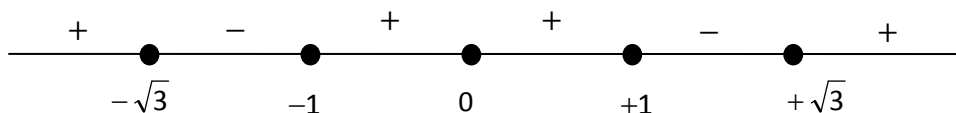
$$x = -1 \Rightarrow (-1, 0)$$

$$x = 0 \Rightarrow f(x) = \frac{0-1}{0-3} = \frac{1}{3} \Rightarrow (0, 1/3)$$

c) Rentang/interval tempat  $f(x)$  positif dan rentang tempat  $f(x)$  negatif.

$$x = 0 \Rightarrow f(x) = \frac{1}{3} > 0 \Rightarrow \begin{array}{ll} f(x) > 0 & \text{pada rentang } -1 < x < 1 \\ f(x) < 0 & x < -1 \\ f(x) < 0 & x > 1 \end{array}$$

$$f(x) \text{ asimtotisdi } \begin{array}{l} x = -\sqrt{3} \\ x = \sqrt{3} \end{array} \Leftrightarrow \lim_{x \rightarrow \pm\sqrt{3}} \frac{x^2 - 1}{x^2 - 3} = \pm\infty$$



d) Rentang tempat  $f(x)$  naik dan rentang tempat  $f(x)$  turun.

$$f(x) = \frac{x^2 - 1}{x^2 - 3}$$

$$f'(x) = \frac{2x(x^2 - 3) - 2x(x^2 - 1)}{(x^2 - 3)^2} = -\frac{4x}{(x^2 - 3)^2}$$

$f'(x) > 0$  jika  $x < 0 \Rightarrow f(x)$  naik  
 $f'(x) < 0$  jika  $x > 0 \Rightarrow f(x)$  turun

e) Titik-titik maksimum dan minimum relatif  $f(x)$ .

$f(x)$  maximum atau minimum untuk  $f'(x) = 0$

$$f(x) = \frac{x^2 - 1}{x^2 - 3}$$

$$f'(x) = \frac{-4x}{(x^2 - 3)^2} = 0 \text{ terjadi di } x = 0$$

$f(x)$  maximum di  $x = 0$ ,  $f(x) = \frac{0 - 1}{0 - 3} = \frac{1}{3}$

f) Rentang tempat kurva  $f(x)$  cembung ke bawah dan cekung ke bawah, serta koordinat titik baliknya.

$$f'(x) = \frac{-4}{(x^2 - 3)^2}$$

$$f''(x) = \frac{-4(x^2 - 3)^2 + (4x)2(x^2 - 3)2x}{(x^2 - 3)^4} = \frac{-4(x^2 - 3)^2 + 4(x^2 - 3)4x^2}{(x^2 - 3)^4}$$

$$= \frac{4(x^2 - 3)(-x^2 + 3 + 4x^2)}{(x^2 - 3)^4} = \frac{4(x^2 - 3)(3x^2 + 3)}{(x^2 - 3)^4}$$

$$= \frac{12(x^2 - 3)(x^2 + 1)}{(x^2 - 3)^4} = \frac{12(x^2 + 1)}{(x^2 - 3)^3}$$

$f''(x) > 0$  jika  $(x^2 - 3)^3 > 0$   
 $x^2 - 3 > 0$   
 $x^2 > 3$   
 $x > \sqrt{3}$  atau  $x < -\sqrt{3}$

Jadi  $f(x)$  cembung ke bawah jika  $x > \sqrt{3}$  atau  $x < -\sqrt{3}$ .

$$f''(x) < 0 \text{ jika } (x^2 - 3)^3 < 0$$

$$x^2 - 3 < 0$$

$$x^2 < 3$$

$$-\sqrt{3} < x < \sqrt{3}$$

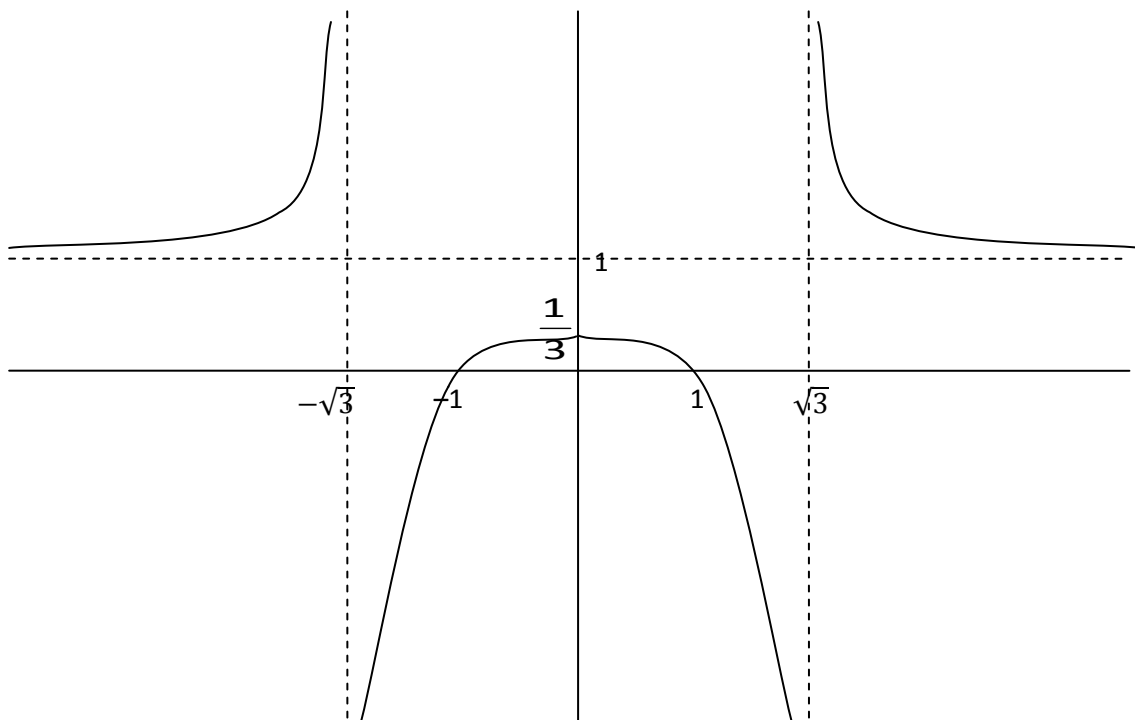
Jadi  $f(x)$  cekung ke bawah jika  $-\sqrt{3} < x < \sqrt{3}$ .

g) Persamaan asimtot  $f(x)$ .

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 - 1}{x^2 - 3} = \lim_{x \rightarrow \pm\infty} \frac{1 - \frac{1}{x^2}}{1 - \frac{3}{x^2}} = \frac{1 - 0}{1 - 0} = 1$$

$$\lim_{x \rightarrow \pm\sqrt{3}} \frac{x^2 - 1}{x^2 - 3} = \pm\infty$$

h) Gambar kurva  $f(x)$ .



4) Carilah integral tak tentu di bawah ini.

a)  $\int (x^3 + \sqrt{x}) dx = \frac{1}{4}x^4 + \frac{2}{3}x\sqrt{x} + c$

b)  $\int (x^2 - 3x + 2)^2 (2x - 3) dx = \int (x^2 - 3x + 2)^2 d(x^2 - 3x + 2) = \frac{1}{3}(x^2 - 3x + 2)^3 + c$

5) Carilah  $f(x)$  jika diketahui  $f''(x) = d^2f(x)/dx^2$  di bawah ini.

a)  $f''(x) = x^{4/3}$

$$f'(x) = \frac{3}{7}x^{7/3} + c$$

$$\begin{aligned} f(x) &= \frac{3}{7} \frac{3}{10} x^{10/3} + cx + c_1 \\ &= \frac{9}{70} x^{10/3} + cx + c_1 \end{aligned}$$

b)  $f''(x) = \frac{x^4 + 1}{x^3} = x + x^{-3}$

$$f'(x) = \frac{1}{2}x^2 - \frac{1}{2}x^{-2} + c$$

$$\begin{aligned} f(x) &= \frac{1}{6}x^3 + \frac{1}{2}x^{-1} + cx + c_1 \\ &= \frac{1}{6}x^3 + \frac{1}{2x} + cx + c_1 \end{aligned}$$

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