

BIVARIATE PROBABILITY DISTRIBUTION

CONTOH

Tabel di bawah ini menunjukkan jumlah event (hari) yang memiliki temperatur rata-rata harian (T) dan kelembaban relatif rata-rata (H) yang dipetik dari data harian periode 8 hari selama 43 tahun.

		Temperature, T (°F)					
		20–30	30–40	40–50	50–60	60–70	70–80
Relative Humidity, H (%)	0–20	2	4	6	2	2	1
	20–40	4	8	12	30	6	9
	40–60	5	15	30	60	30	20
	60–80	3	7	9	25	17	11
	80–100	1	0	2	12	8	3

Carilah:

- $f_{T,H}(t_i, h_j)$
- $f_T(t_i)$ dan $F_T(t_i)$
- $f_H(h_j)$ dan $F_H(h_j)$
- Probability:
 - $40 \leq T \leq 50$ dan $60 \leq H \leq 80$
 - $40 \leq T \leq 50$ pada saat $60 \leq H \leq 80$
 - $T \leq 60$
 - $H \leq 60$
 - $T \leq 40$ dan $H \leq 40$
- Apakah T dan H independent?

PENYELESAIAN

Temperatur: $T \rightarrow 6$ interval: $t_i, i = 1, 2, \dots, 6$

Kelembaban: $H \rightarrow 5$ interval: $h_j, j = 1, 2, \dots, 5$

$n_{i,j}$ = jumlah hari sesuai dengan interval i, j

$$N = \sum_{i,j} n_{i,j} = 344$$

$$f_{T,H}(t_i, h_j) = \frac{n_{i,j}}{N}$$

Hitungan frekuensi setiap nilai temperatur dan kelembaban relatif disajikan pada tabel di bawah ini.

j	i						$f_H(h_j)$	$F_H(h_j)$
	1	2	3	4	5	6		
1	0.0058	0.0116	0.0174	0.0058	0.0058	0.0029	0.0494	0.0494
2	0.0116	0.0233	0.0349	0.0872	0.0174	0.0262	0.2006	0.2500
3	0.0145	0.0436	0.0872	0.1744	0.0872	0.0581	0.4651	0.7151
4	0.0087	0.0203	0.0262	0.0727	0.0494	0.0320	0.2093	0.9244
5	0.0029	0.0000	0.0058	0.0349	0.0233	0.0087	0.0756	1.0000
$f_T(t_i)$	0.0436	0.0988	0.1715	0.3750	0.1831	0.1279		
$F_T(t_i)$	0.0436	0.1424	0.3140	0.6890	0.8721	1.0000		

Joint and conditional probabilities (untuk discrete random variables)

– $\text{prob}(40 \leq T \leq 50 \text{ dan } 60 \leq H \leq 80) = f_{T,H}(t_3, h_4) = 0.0262$

– $\text{prob}(40 \leq T \leq 50 \text{ pada saat } 60 \leq H \leq 80) = f_{T,H}(t_3|h_4)$

$$f_{T,H}(t_3|h_4) = \frac{f_{T,H}(t_3, h_4)}{f_H(h_4)}$$

$$= \frac{0.0262}{0.2093} = 0.125$$

– $\text{prob}(T \leq 60) = F_T(60) = F_T(t_4) = 0.6889$

– $\text{prob}(H \leq 60) = F_H(60) = F_H(h_3) = 0.7151$

– $\text{prob}(T \leq 40 \text{ dan } H \leq 40) = F_{T,H}(t_2, h_2)$

$$F_{T,H}(t_2, h_2) = \sum_{i=1}^2 \sum_{j=1}^2 f_{T,H}(t_i, h_j) = 0.0522$$

– Jika T dan H independent, maka $f_T(t_i) \cdot f_H(h_j) = f_{T,H}(t_i, h_j)$

$i = 3$ dan $j = 3$:

$$\left. \begin{array}{l} f_T(t_3) = 0.1715 \\ f_H(h_3) = 0.4651 \end{array} \right\} f_{T,H}(t_3, h_3) = 0.0872 \Rightarrow f_T(t_3) \cdot f_H(h_3) \neq f_{T,H}(t_3, h_3)$$

dengan demikian T dan H dependent.