UNIVERSITAS GADJAH MADA
DEPARTEMEN TEKNIK SIPIL DAN LINGKUNGAN
PRODI MAGISTER TEKNIK SIPIL

Semester I

Dinamika Aliran dan Transfer Massa

Materi Kuliah

Minggu ke-1 s.d. ke-8	Aliran air di sungai
	Persebaran polutan di sungai
	Persebaran air lindi ke air tanah
Minggu ke-9 s.d. ke-16	Transfer massa dalam satu fasa
	Transfer massa antar fasa
	Tipe reaktor

Agenda Kuliah

Minggu ke-	Topik	Materi
1	• Pengantar kuliah. • Persebaran polutan di sungai. • Jenis aliran.	• Rencana kegiatan pembelajaran. • Evaluasi belajar, bobot nilai (PR, Kuis, UTS, UAS). • Contoh kasus pencemaran sungai. • Aliran air: aliran muka air bebas, aliran air di pipa, aliran air tanah. • Aliran turbulen, aliran laminar. • Aliran permanen, aliran tak permanen. • Aliran subkritis, aliran kritis, aliran superkritis. • Aliran air, aliran air+sedimen, aliran air+polutan.
2	Persebaran polutan di sungai.	• Mekanisme persebaran polutan (transpor massa) di sungai: difusi, konveksi-difusi, konveksi-difusi di aliran turbulen. • Persamaan matematik persebaran polutan. • Penyelesaian analitis persamaan difusi 1D.

Agenda Kuliah

Minggu ke-	Topik	Materi
3	Persebaran polutan secara difusi (1/2).	• Praktik hitung persebaran difusif polutan 1D. • Difusi 1D di medium berbatas. • Praktik hitung difusi 1D di medium berbatas.
4	Persebaran polutan secara difusi (2/2).	• Praktik hitung difusi 1D dari sumber menerus. • Penyelesaian analitis persamaan difusi 2D. • Penyelesaian analitis persamaan difusi 3D.
5	Persebaran polutan secara konveksi-difusi (1/2).	• Persamaan konveksi-difusi di sungai. • Koefisien difusi (vertikal, transversal, longitudinal, rerata kedalaman). • Dispersi *vs* difusi molekular. • Praktik hitung persebaran polutan konvektif-difusif.
6	Persebaran polutan secara konveksi-difusi (2/2).	• Praktik hitung persebaran polutan konvektif-difusif (lanjutan). • Praktik hitung persebaran polutan konvektif-difusif di medium berbatas.
7	Pengantar model persebaran polutan.	• Penyelesaian persamaan polutan secara numerik *vs* secara analitik. • Pengenalan beberapa *software* untuk memodelkan persebaran polutan.
8	UTS	Pengerjaan soal UTS dilakukan di kelas dan di luar kelas.

istiarto

Departemen Teknik Sipil dan Lingkungan Fakultas Teknik UGM

Dinamika Aliran dan Transfer Massa

Mata kuliah Dinamika Aliran dan Transfer Massa adalah salah satu mata kuliah wajib di Prodi Magister Teknik Sipil, Minat Lingkungan. Bobot mata kuliah adalah 3 sks.

Materi Kuliah

1. Aliran air di sungai
2. Persebaran polutan di sungai
3. Persebaran air lindi ke air tanah
4. Transfer massa dalam satu fasa
5. Transfer massa antar fasa

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Dinamika Aliran dan Transfer Massa

Permasalahan Cemaran Air Sungai

Pencemaran Air Sungai

- Kasus sungai tercemar polutan pada masa lalu
  - Sungai Songhua, China, November 2005
  - Sungai Danube, Eropa, Oktober 2010

TRAGEDI HARBIN

Ikan dari Sungai Songhua Dikonsumsi Warga

HARBIN, JUMAT — Jutaan warga kota Harbin, China, bertahan hingga hari ketiga tanpa air. Aliran air sepanjang 80 kilometer di Sungai Songhua yang tercemar zat kimia benzena melewati kota Harbin dengan sangat pelan karena udara dingin yang membuat air sungai itu agak membeku.

Meski demikian, arus pengungsian dari kota Harbin juga terlihat karena khawatir dampak dari pencemaran Sungai Songhua itu akan bertahan lama.

Sementara itu, para ahli teknik China juga terus bekerja keras menggali puluhan sumur untuk menjamin terpenuhinya kebutuhan air bagi 3,8 juta warga kota Harbin, yang merupakan kota industri di Provinsi Heilongjiang.

Meski sudah berlangsung beberapa hari, informasi mengenai tercemarnya Sungai Songhua tidak menyebar cukup luas di kalangan warga di sepanjang sungai tersebut. Penduduk desa-desa dan kota yang berada di sepanjang Sungai Songhua, antara Jilin dan Harbin, kemarin mengaku masih mengonsumsi ikan dari sungai tersebut karena tidak ada yang memberi tahu mereka mengenai tercemarnya sungai itu.

Ketika diberi tahu bahwa ikan-ikan tersebut berbahaya untuk dikonsumsi, warga pun kaget dan langsung membuang ikan yang mereka beli di pasar itu.

PBB tawarkan bantuan

Kemarin PBB juga telah menawarkan kepada China untuk membantu penanganan pencemaran zat kimia di sungai yang terus mengalir ke Rusia itu, tetapi Pemerintah China belum merespons tawaran bantuan tersebut.

"Kami membutuhkan informasi dasar, informasi resmi dari Pemerintah China, yang kami belum punyai," kata Vladimir Sakharov, Kepala Bidang Tanggap Darurat Lingkungan di Badan Perlindungan Lingkungan PBB (UNEP).

Pemerintah China memperkirakan sekitar 100 ton benzena dan nitrobenzena telah masuk ke Sungai Songhua akibat meledaknya sebuah pabrik kimia di Provinsi Jilin, di hulu Songhua.

GUNAWAN

Meski Pemerintah China berusaha menyampaikan bahwa dampak pencemaran itu tidaklah besar, pakar-pakar lingkungan kemarin memperingatkan bahwa dampak lingkungan dan sosial akibat pencemaran itu bisa berlangsung bertahun-tahun.

Sebagian zat benzena akan mengendap di lapisan dasar sungai dan masuk ke dalam rantai makanan di sungai tersebut. "Mereka yang berada di bagian atas rantai makanan, seperti burung-burung air dan manusia, bisa merasakan akibatnya," ujar Kenneth Leung, seorang ekotoksikolog dari Universitas Hongkong, sambil menambahkan bahwa benzena itu di dalam tubuh manusia bisa bermutasi, yang kemudian menyebabkan kanker.

Sementara di wilayah Rusia, kekhawatiran warga pun semakin meningkat karena air tercemar itu akan segera mencapai perbatasan Rusia dan masuk ke wilayah Rusia. Kremlin kemarin mengumumkan telah melakukan sejumlah langkah darurat untuk melindungi jutaan warganya.

Ledakan kedua

Belum lagi dampak ledakan pabrik kimia di Jilin bisa ditangani, sebuah pabrik kimia lainnya telah meledak di tenggara China, menewaskan seorang pekerja dan memaksa 10.000 penduduk di sekitarnya segera diungsikan.

Media China kemarin melaporkan, ledakan terjadi hari Kamis (24/11) di pabrik kimia Yingte di Chongqing, Provinsi Dianjiang.

Pejabat setempat segera melarang warga menggunakan air di sungai sekitarnya, sementara pengujian atas air dilakukan. Namun dilaporkan zat beracun tidak sampai mencemari lingkungan sekitarnya akibat ledakan tersebut. (AFP/AP/BBC/OKI)

KOMPAS, SABTU, 26 NOVEMBER 2005 8

EPA/NIU YIXIN

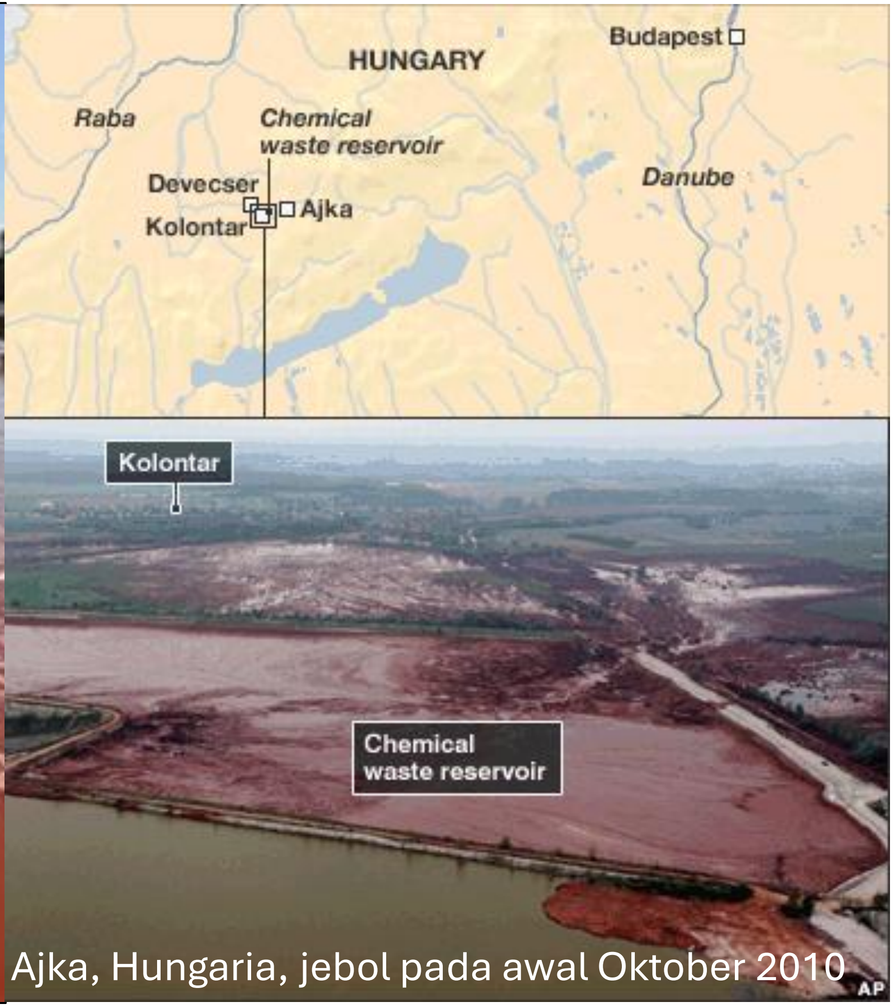
Aliran Sungai Songhua juga melewati kota Harbin. Air yang terkontaminasi bahan kimia (benzena) mencapai Harbin pada Kamis (24/11). Bahan kimia penyebab kanker itu bersumber dari ledakan di sebuah pabrik kimia yang terletak di hulu sungai pada 13 November lalu. Pasokan air dihentikan untuk sementara di Harbin.

TRAGEDI HARBIN

Ikan dari Sungai Songhua Dikonsumsi Warga

Harbin’s Disaster

- More stories on Harbin’s Songhua River pollution in November 2005
  - http://www.gov.cn/english/2005-11/25/content_108891.htm
  - webarchive on Harbin’s Songhua River pollution in November 2005



Penampungan limbah di sebuah pabrik kimia di Ajka, Hungaria, jebol pada awal Oktober 2010


Vienna
0
25
50km
AUSTRIA
SLOVAKIA
Mosoni-Danube
Gyor
Danube
Affected waters
Raba
HUNGARY
Budapest
Marcal
Chemical waste reservoir
Ajka
Kolontar
SLOVENIA

Danube River Disaster

- More stories on Danube River pollution in October 2010
  - http://www.bbc.co.uk/news/world-europe-11495540
  - webarchive file
  - http://www.guardian.co.uk/world/2010/oct/12/danube-toxic-soviet-hungary-sludge
  - webarchive file

Dinamika Aliran dan Transfer Massa

Aliran Air di Sungai

Aliran air

- Aliran air di saluran terbuka (*open channel flow*)
- Aliran air di pipa (*pipe flow*)
- Aliran air tanah (*ground water flow*)

Open channel flow

- Muka air bebas
- Muka air bertekanan atmosferik

Pipe flow

- Aliran bertekanan lebih besar daripada tekanan atmosfir

Ground water flow

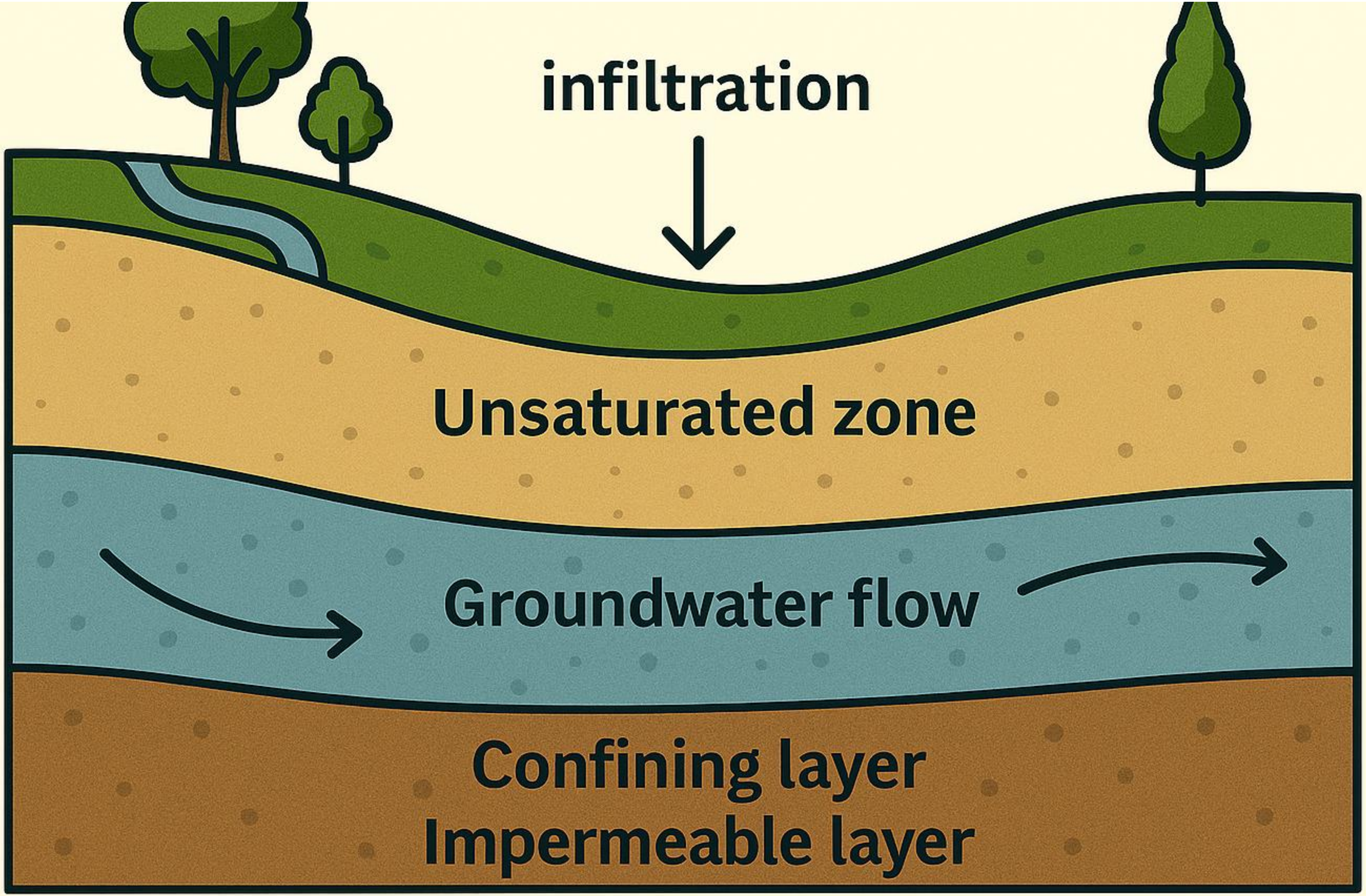
- Aliran di bawah muka tanah


open channel flow → aliran di sungai
tekanan atmosferik
AI generated image (10Aug2025)


open channel flow di gorong-gorong (*ai generated image*)

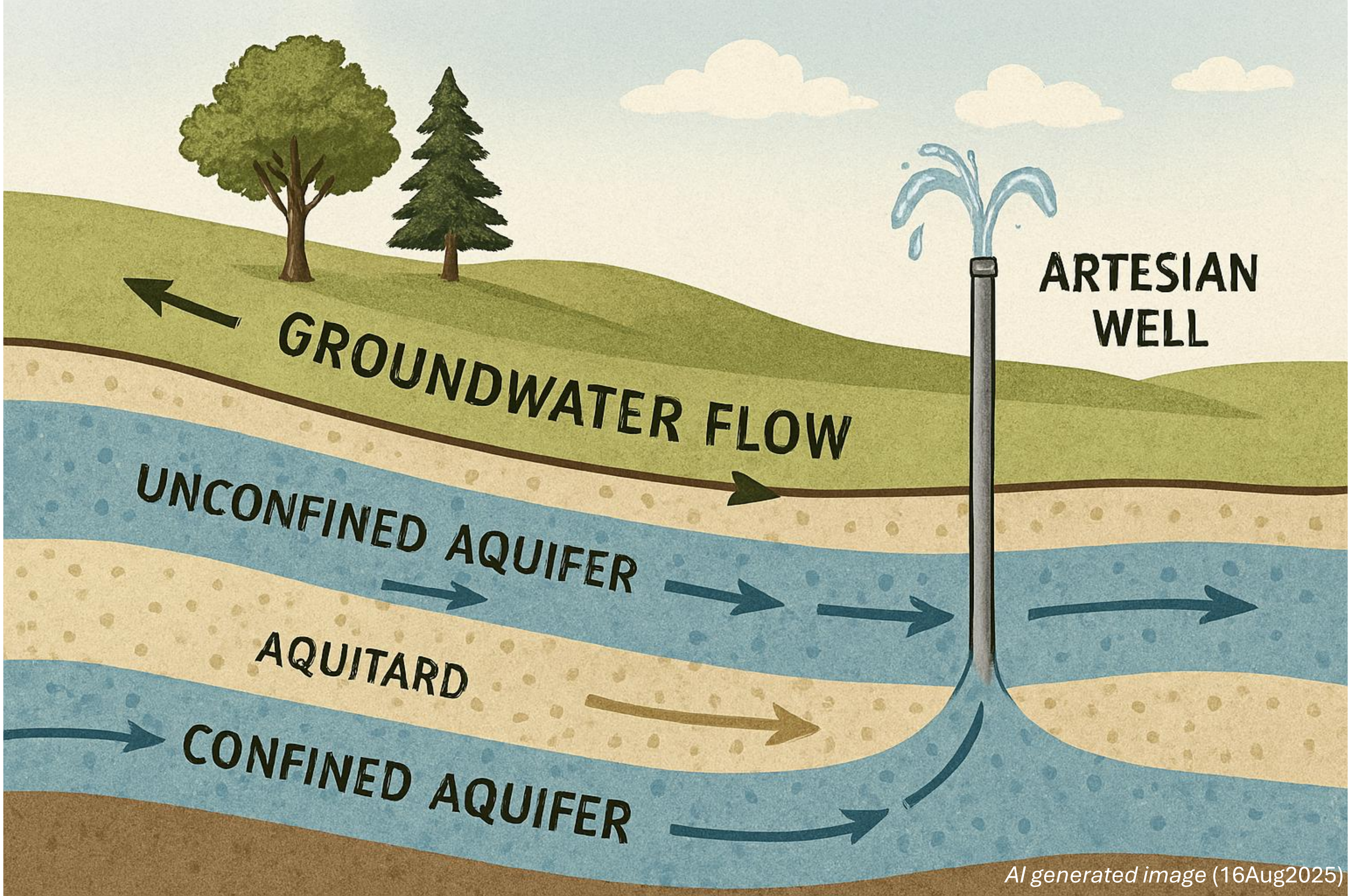

open channel flow di terowongan (*ai generated image*)

- Aliran di saluran terbuka (*open channel flow*)
  - mengacu pada muka air, yaitu muka air bertekanan atmosferik
  - bukan mengacu pada bentuk atau jenis saluran
  - aliran di saluran tertutup (terowongan, gorong-gorong, pipa) namun muka air bertekanan atmosferik adalah *open channel flow*



AI generated image (16Aug2025)

- Air tanah
  - Air tanah di *unconfined aquifer*
    - Air tanah di lapis tak jenuh (*unsaturated zone*)
      - Aliran air tanah bertekanan negatif
    - Air tanah di lapis tanah jenuh (*saturated zone*)
      - Muka air tanah bertekanan atmosferik
  - Air tanah di *confined aquifer*
    - →
      - Aliran air tanah bertekanan



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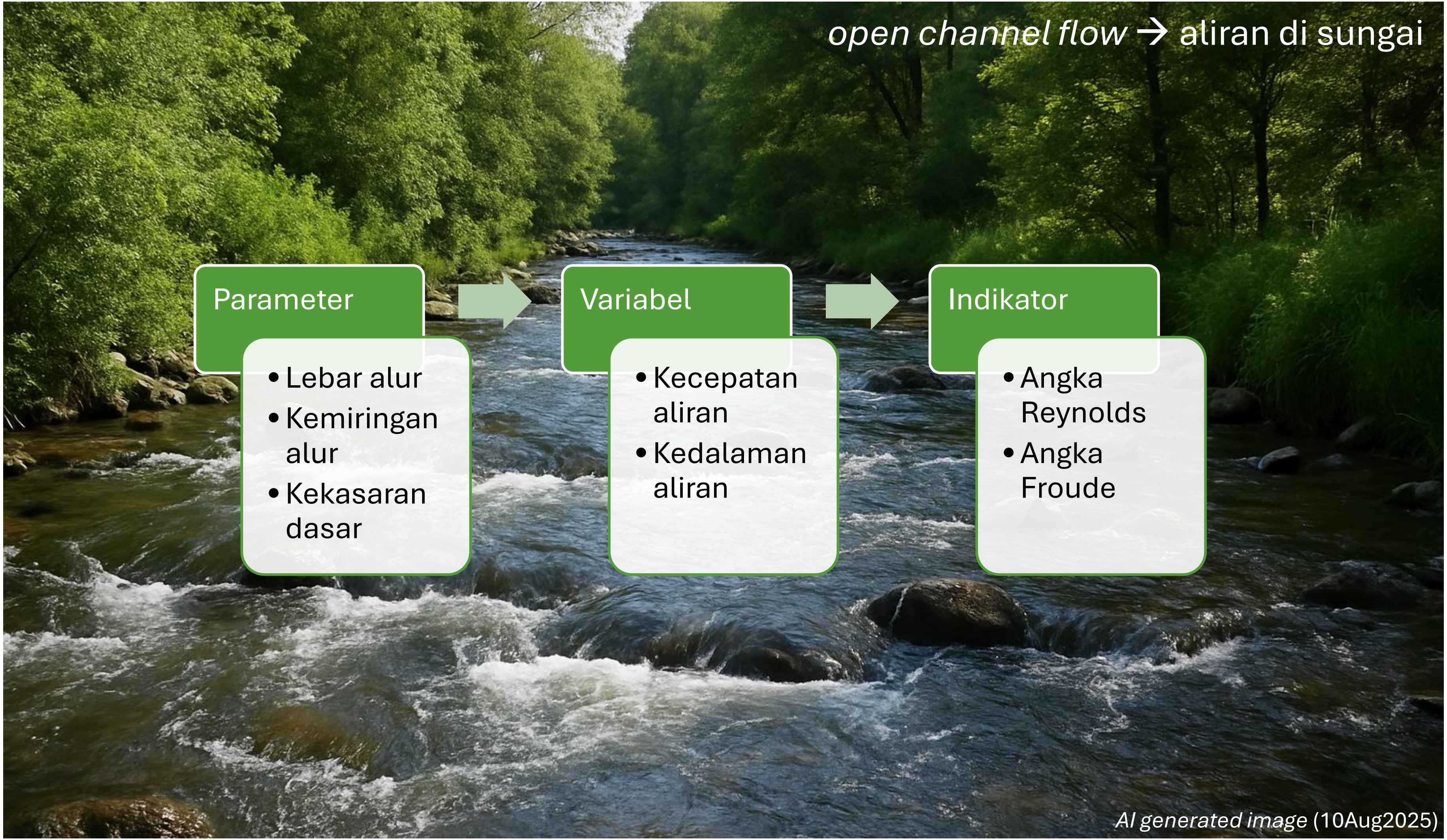

open channel flow → aliran di sungai

Parameter
- Lebar alur
- Kemiringan alur
- Kekasaran dasar

→

Variabel
- Kecepatan aliran
- Kedalaman aliran

→

Indikator
- Angka Reynolds
- Angka Froude

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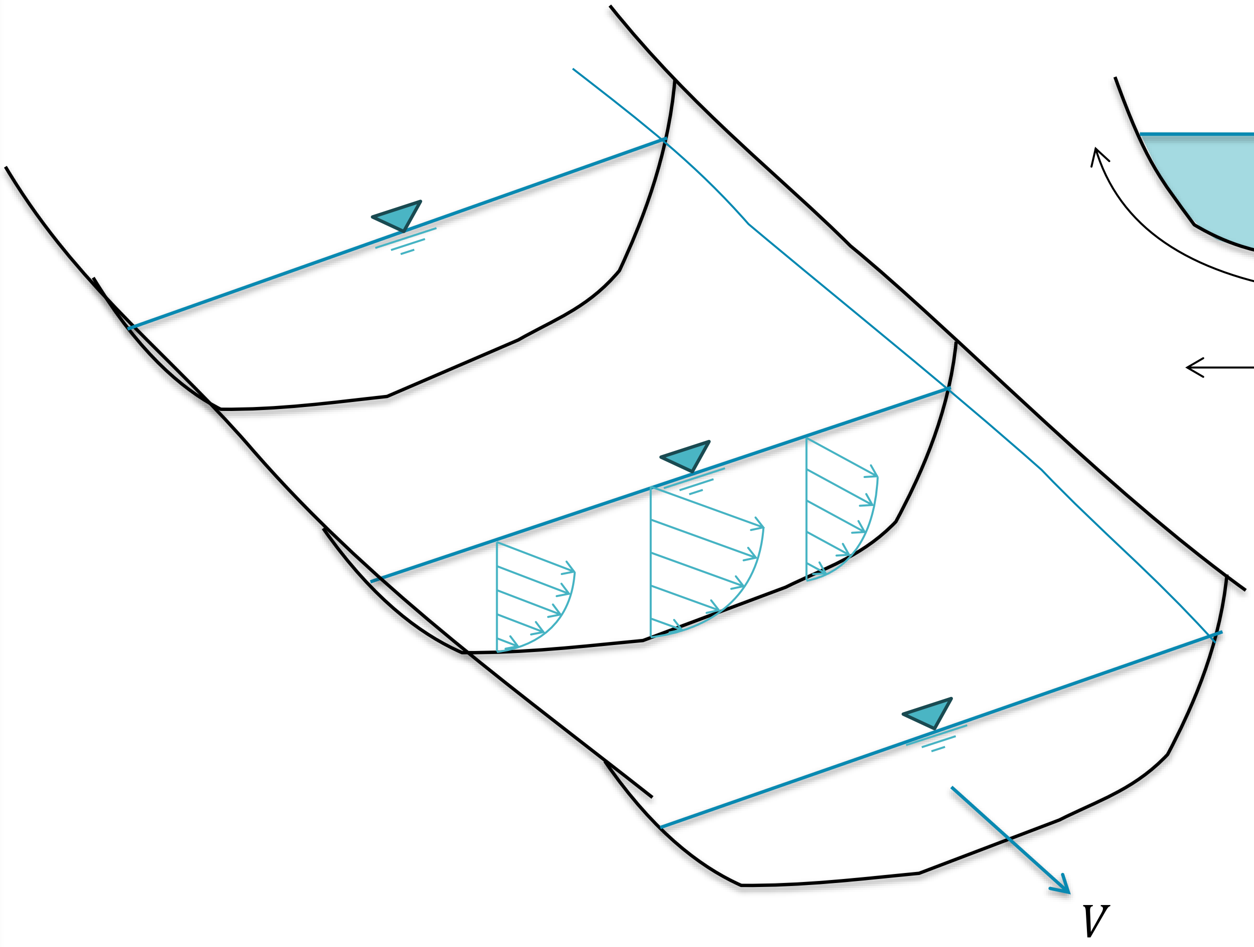
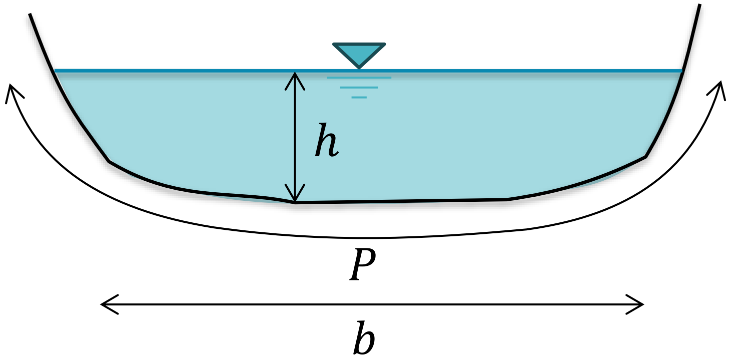




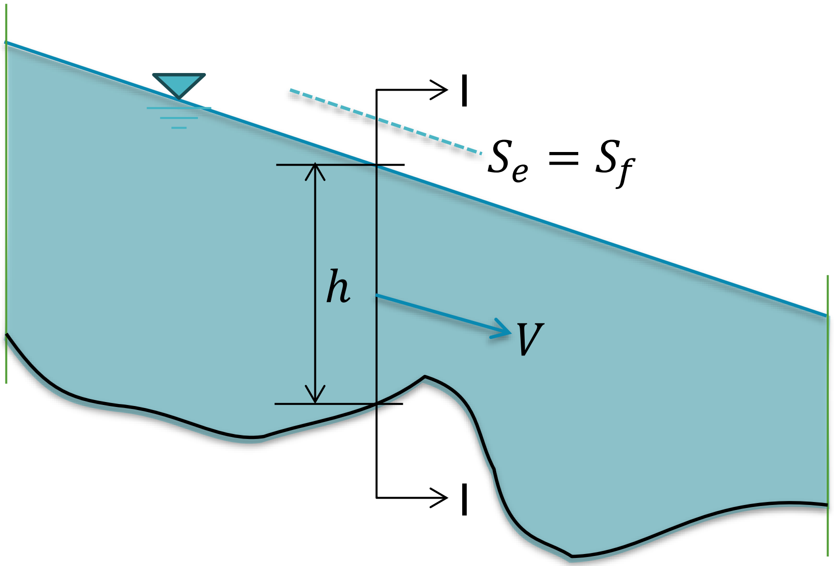
b lebar tampang aliran

h kedalaman aliran

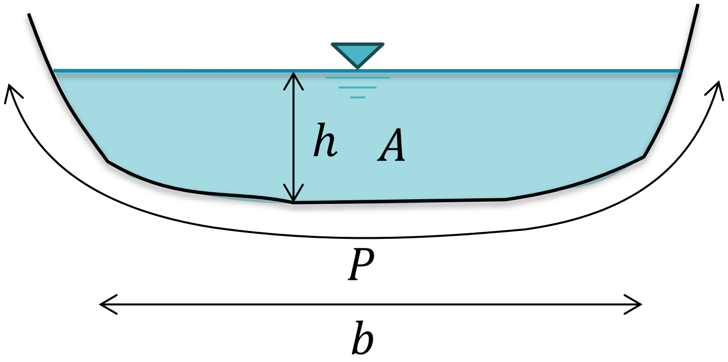
P keliling basah tampang aliran

$R_h = A/P$ radius hidraulik

V kecepatan-aliran rerata tampang



Potongan I-I



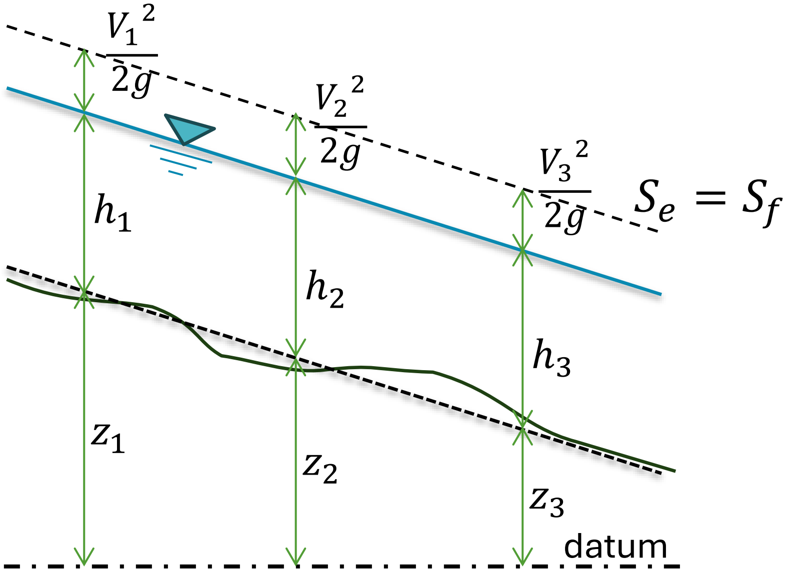
b lebar tampang aliran
h kedalaman aliran
P keliling basah tampang aliran
A luas tampang aliran
$R_h = A/P$ radius hidraulik
V kecepatan-aliran rerata tampang
S_e, S_f kemiringan garis energi akibat friksi aliran dengan kekasaran dasar

$$V = \frac{1}{n} R_h^{\frac{2}{3}} S_f^{\frac{1}{2}}$$

Persamaan Manning berlaku untuk **aliran seragam**

$$Q = VA$$

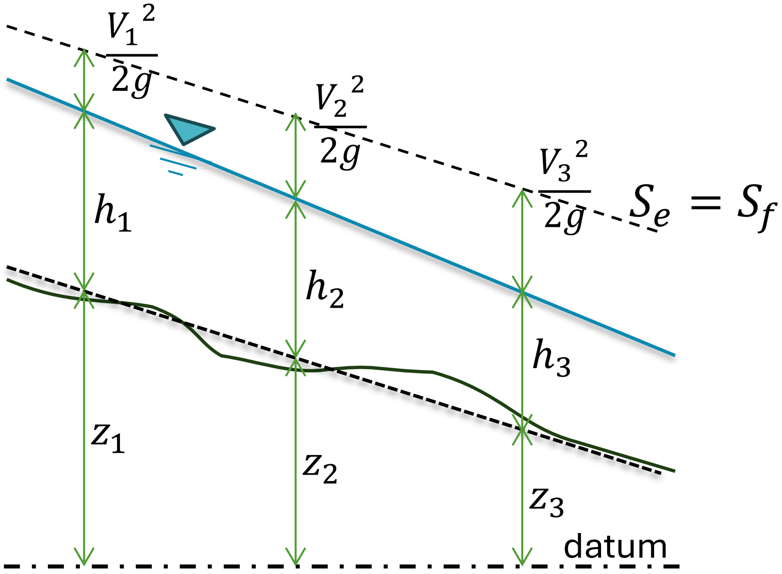
n koefisien kekasaran dasar Sungai, koefisien Manning
Q debit aliran



Aliran seragam (*uniform flow*)

$h_1 = h_2 = h_3$

$V_1 = V_2 = V_3$

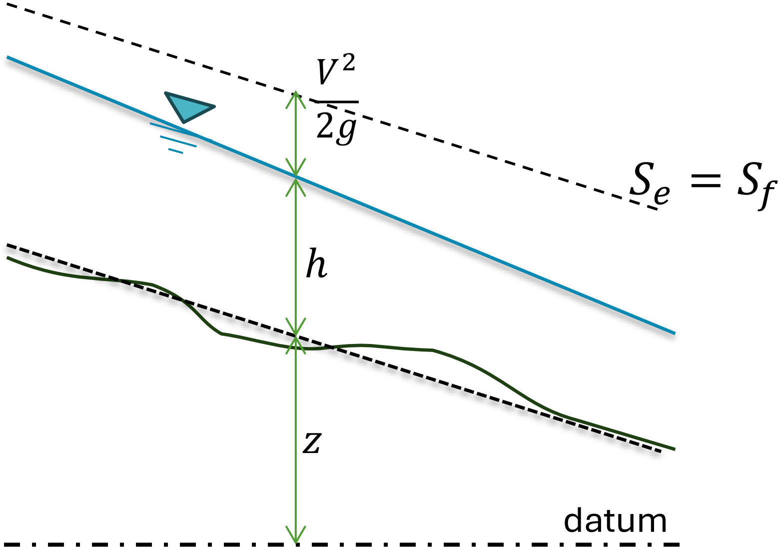


Aliran tak seragam (*nonuniform flow*)

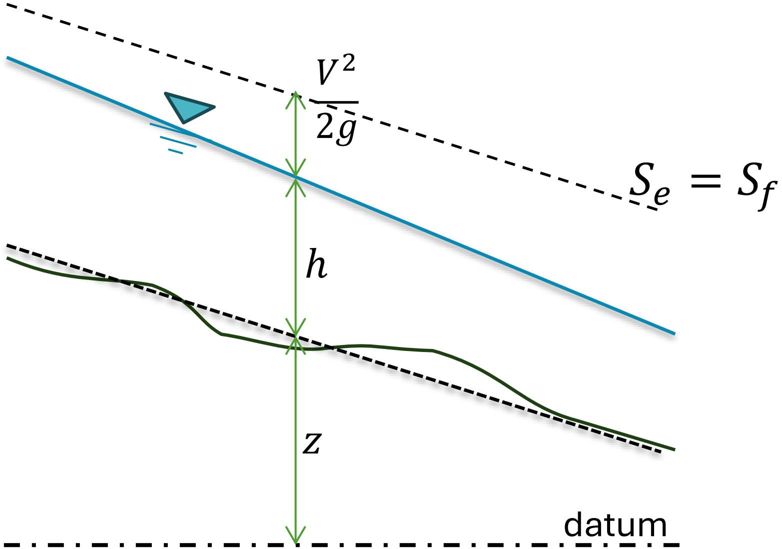
$h_1 \neq h_2 \neq h_3$

$V_1 \neq V_2 \neq V_3$

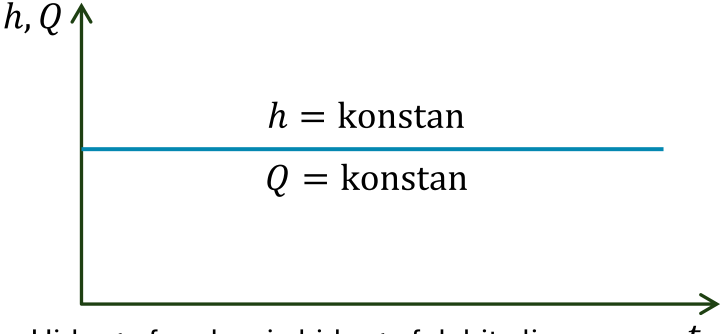
Aliran air di sungai $h_1 \neq h_2 \neq h_3 \Longrightarrow$ **aliran tak seragam**



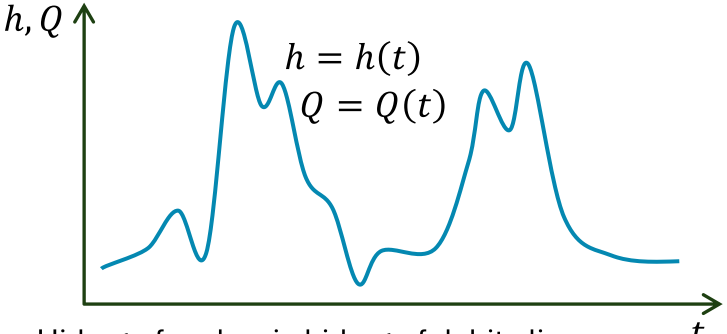
Aliran permanen (*steady flow*)



Aliran tak permanen (*unsteady flow*)



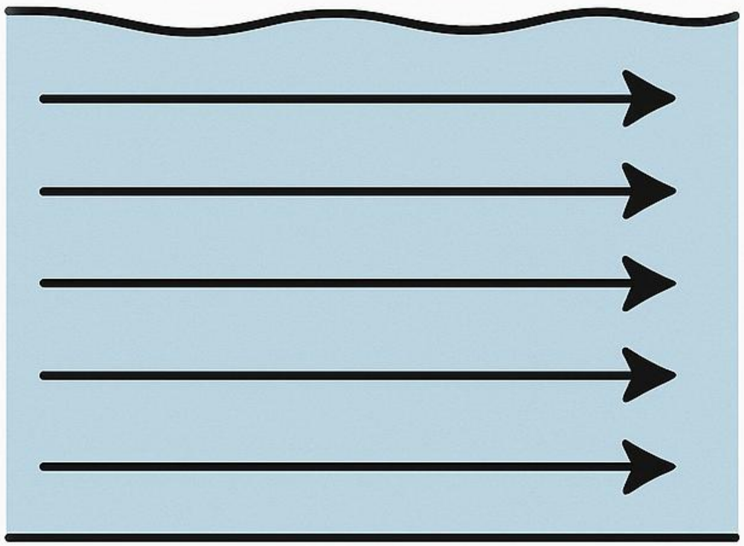
Hidrograf muka air, hidrograf debit aliran



Hidrograf muka air, hidrograf debit aliran

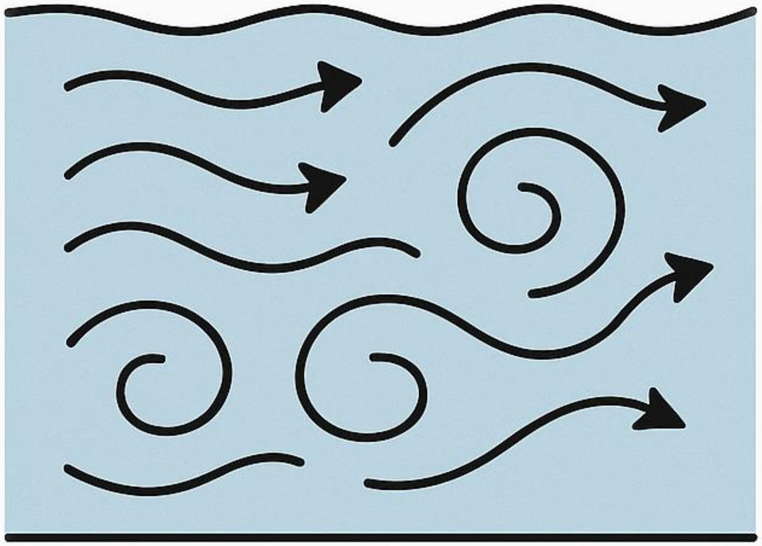
Aliran air di sungai $h = h(t) \Longrightarrow$ **aliran tak permanen**

Aliran laminar



$Re < 500$

Aliran turbulen



$Re > 2000$

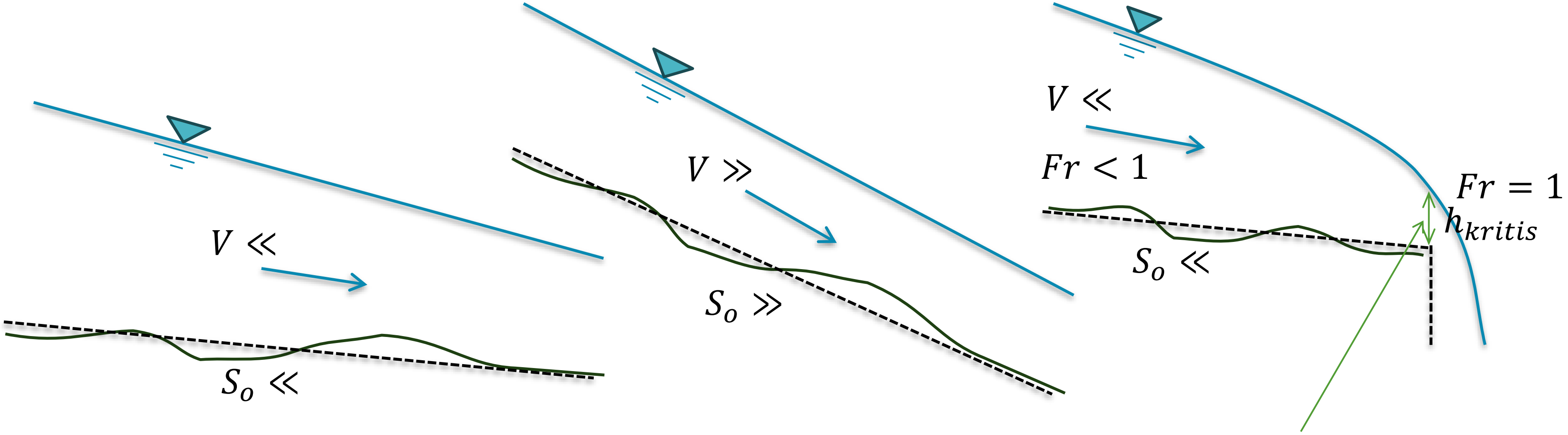
Angka Reynolds

$$Re = \frac{V R_h}{\nu}$$

V kecepatan aliran
R_h radius hidraulik
ν viskositas kinematik air

temperatur (℃)	$\nu_{\text{air}}\ (\times 10^{-6}\ \text{m}^2/\text{s})$
25	0.89
30	0.80

Aliran air di sungai $Re \gg 2000 \Longrightarrow$ **aliran turbulen**



Aliran subkritis, $Fr < 1$

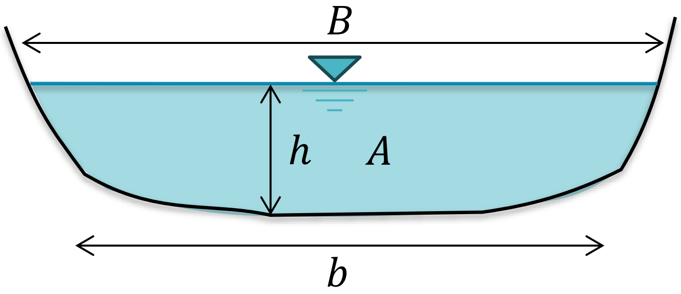
Aliran superkritis, $Fr > 1$

Aliran kritis, $Fr = 1$

Angka Froude

$$Fr = \frac{V}{\sqrt{gD}}$$

D kedalaman hidraulik
$D = A/B$
A luas tampang aliran
B lebar muka air




Buleleng, Sultra, 2019
kritis


Buleleng, Sultra, 2019
superkritis


Outlet terowongan Bendung Segawe ke Waduk Wonorejo, 2022
superkritis

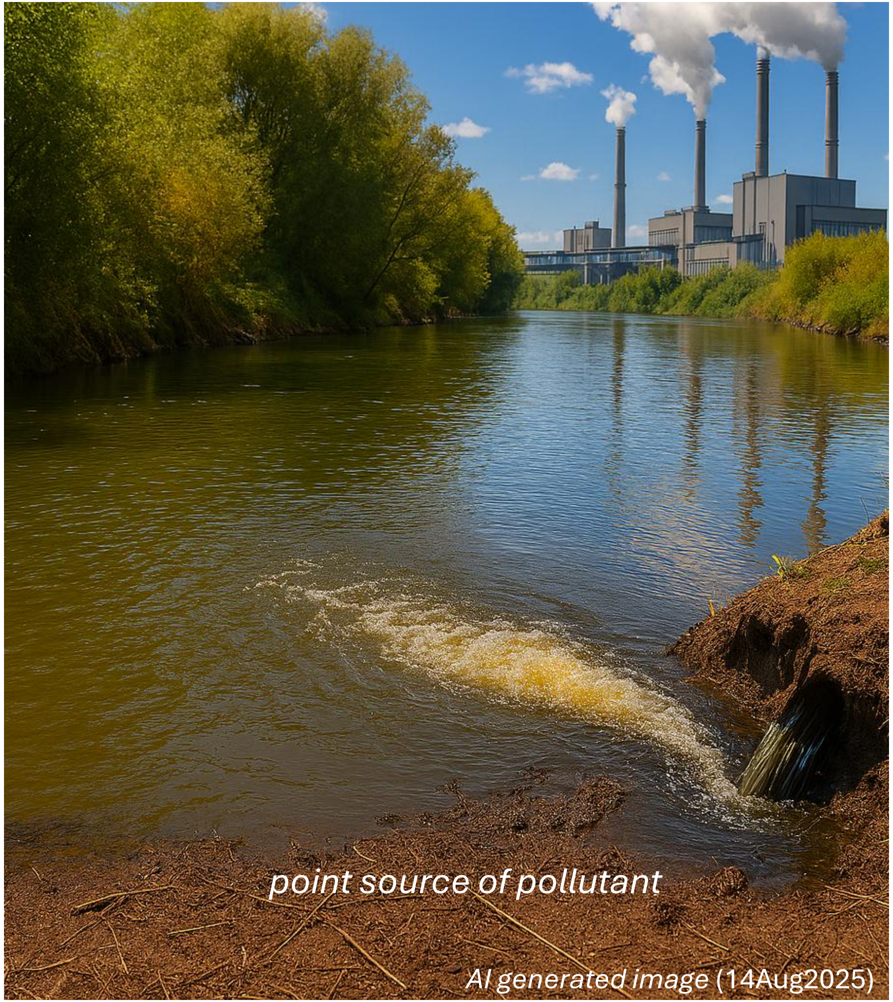

Bendung Colo, 2008
kritis
superkritis

Dinamika Aliran dan Transfer Massa

Persebaran Polutan

Referensi

Graf and Altinakar, 1998, *Fluvial Hydraulics*, Chapter 8, pp. 517-609, J. Wiley and Sons, Ltd., Sussex, England

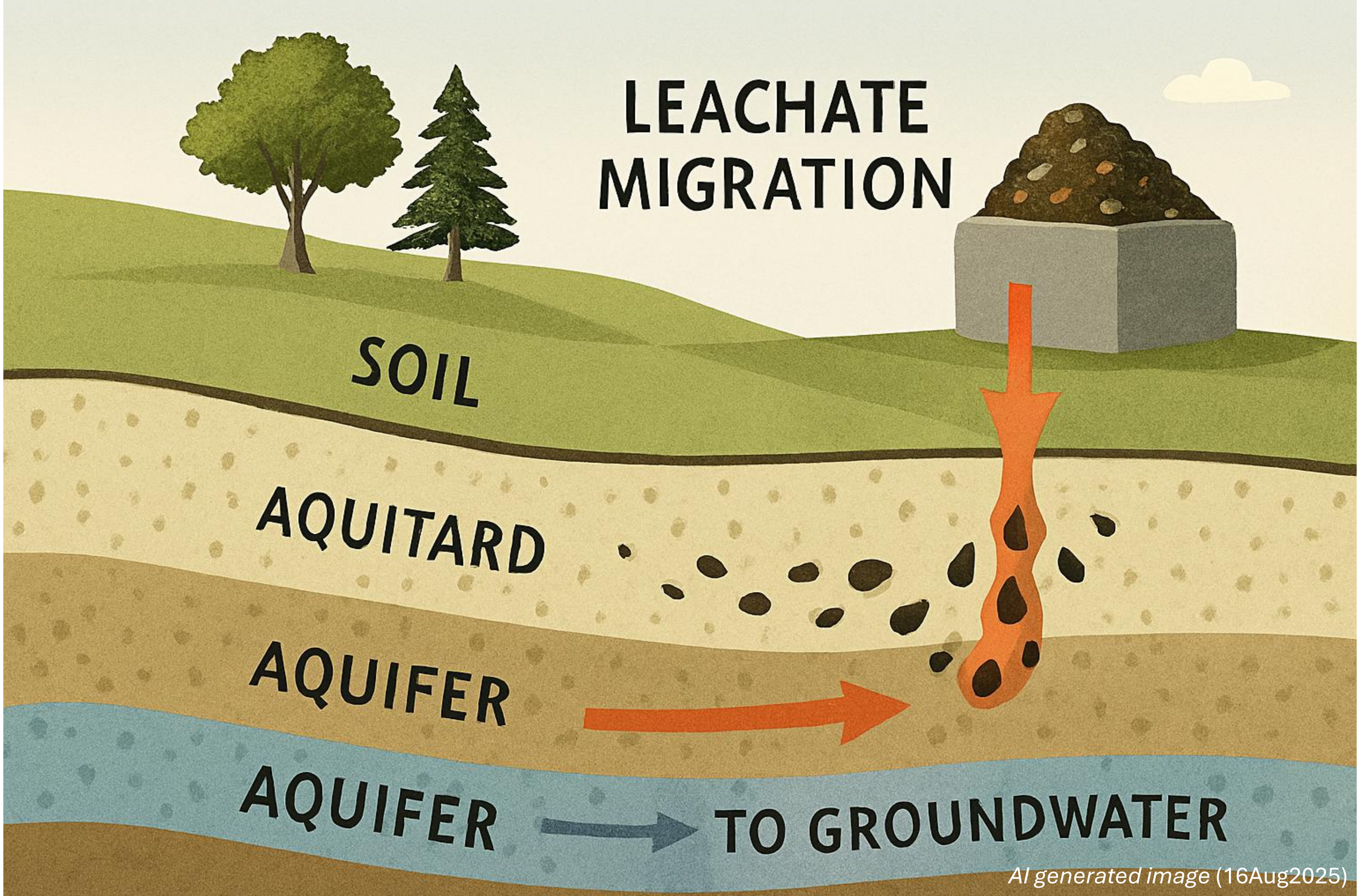

point source of pollutant
AI generated image (14Aug2025)


point source of pollutant
AI generated image (16Aug2025)

Persebaran Polutan

- Mekanisme persebaran polutan di sungai
  - Difusi
    - persebaran yang dipicu oleh perbedaan konsentrasi
    - bergantung pada sifat polutan (koefisien difusi)
  - Konveksi
    - persebaran yang dipicu oleh aliran air

AI generated image (16Aug2025)



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difusi molekular

konveksi laminar dan difusi molekular

konveksi dan difusi dalam aliran turbulen

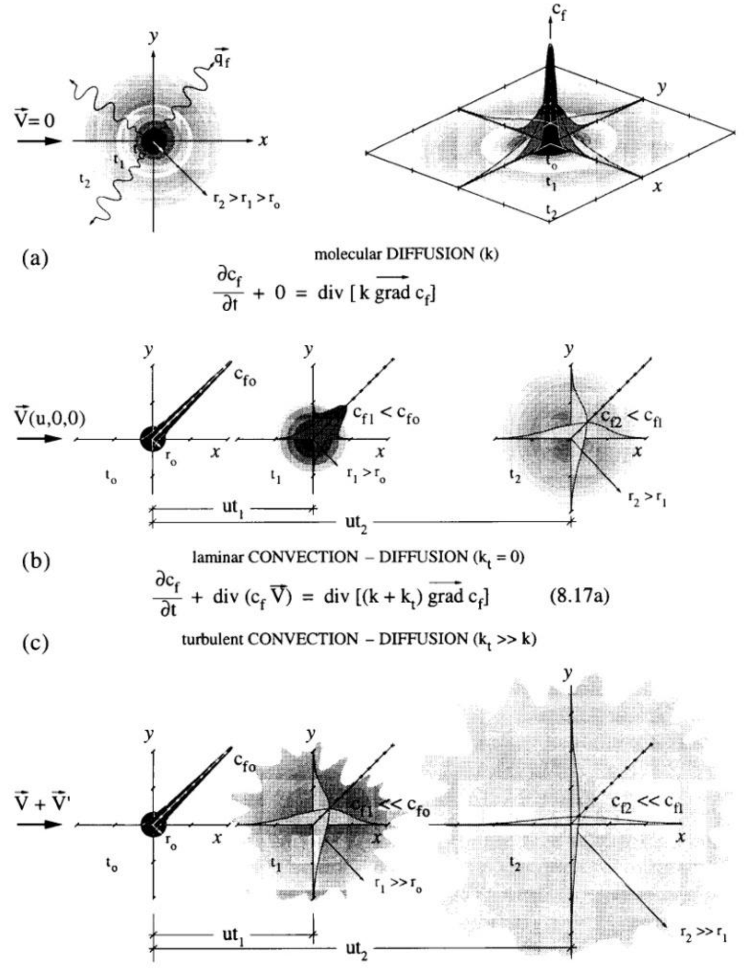
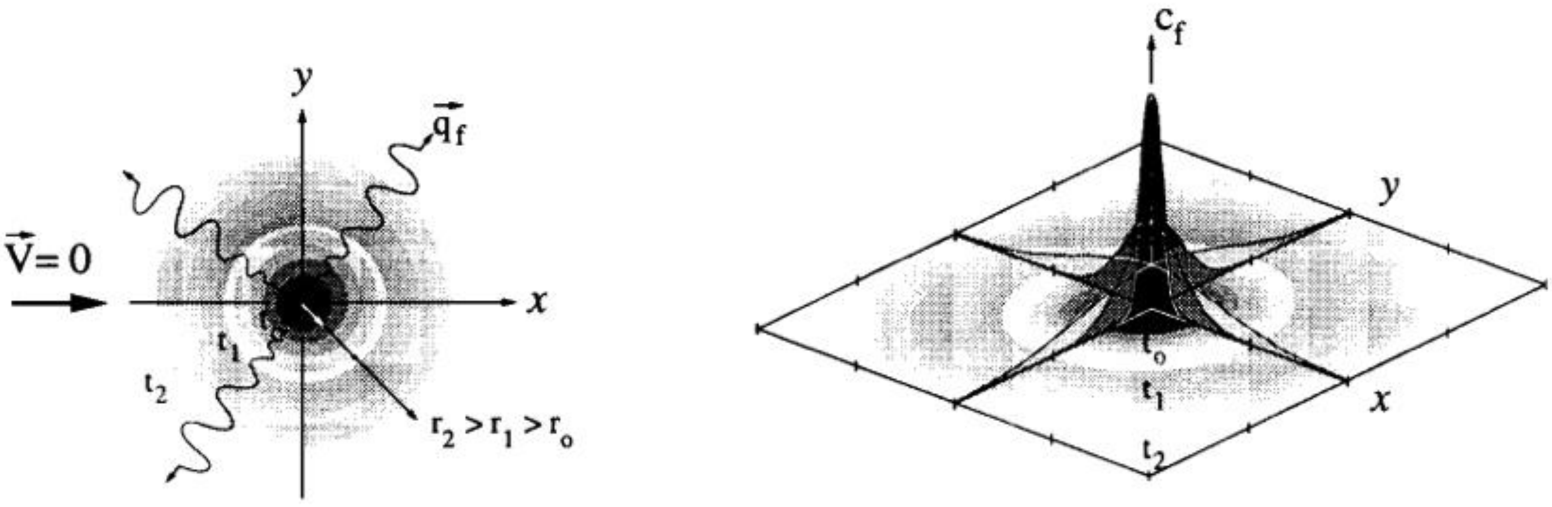


Fig. 8.1 Schematic representation of the *diffusion* and of the *convection-diffusion* in laminar and turbulent regime for a physical property, $c_f(x,y;t)$.

Difusi Molekular

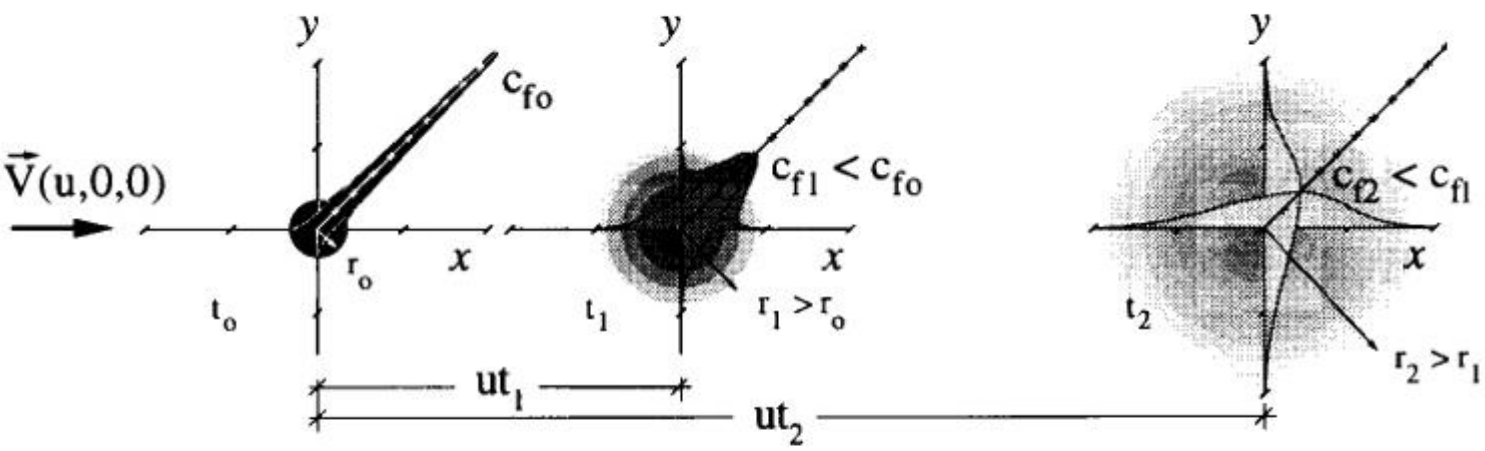


(a)

molecular DIFFUSION (k)

$$\frac{\partial c_f}{\partial t} + 0 = \mathrm{div}\left[k\, \overrightarrow{\mathrm{grad}}\, c_f \right]$$

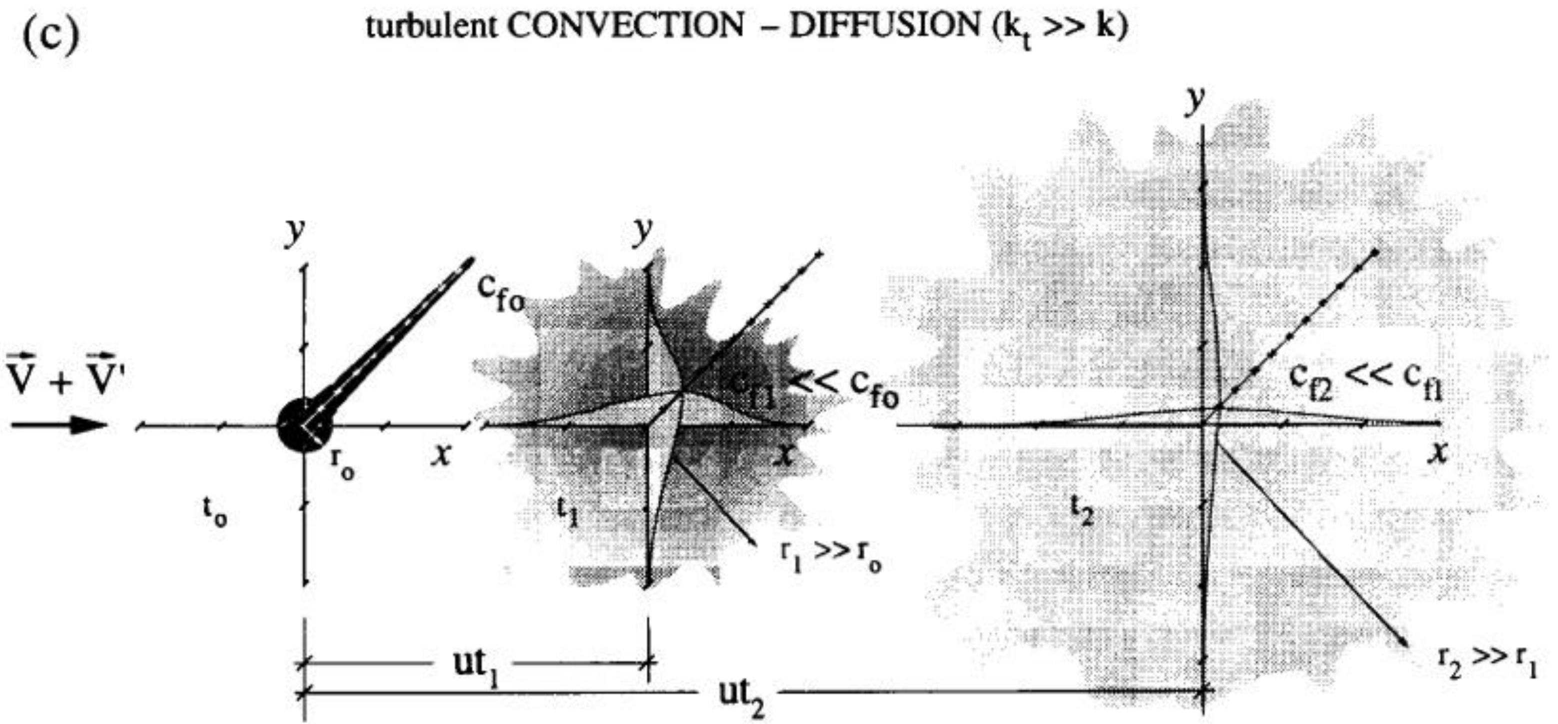
Konveksi Laminar dan Difusi Molekular



(b) laminar CONVECTION – DIFFUSION ($k_t = 0$)

$$\frac{\partial c_f}{\partial t} + \operatorname{div}\left(c_f \vec{V}\right) = \operatorname{div}\left[(k + k_t)\,\overrightarrow{\text{grad}}\,c_f\right] \qquad (8.17a)$$

Konveksi-Difusi dalam Aliran Turbulen



Difusi

- Dalam bahasa matematika, difusi dituliskan sbb.

$$\overrightarrow{q_f} = -k\vec{\nabla}c_f \qquad \overrightarrow{q_f} = -k\,\frac{\partial c_f}{\partial x_i}\vec{i} \qquad \overrightarrow{q_f} = -k\,\overrightarrow{\mathrm{grad}}\,c_f$$

  - k = konstanta = koefisien difusi = *diffusivity*
    - k merupakan parameter karakteristik polutan
    - k bergantung pada temperatur dan tekanan

Difusi

- Sifat proses difusi
  - tidak dapat kembali, setelah menyebar tidak dapat kembali menyatu
  - mengakibatkan kehilangan energi
- Contoh difusi
  - difusi massa
  - difusi termal
  - difusi momentum

Difusi

- Difusi massa → Fick's law $$q_{m,i} = -\epsilon_m \frac{\partial c_f}{\partial x_i}$$
- Difusi termal → Fourier's law $$q_{h,i} = -\rho a_h C_p \frac{\partial T}{\partial x_i} \quad \left(\rho C_p = \mathrm{konstan}\right)$$
- Difusi momentum → Newton's law $$q_{mt,ij} = -\rho \nu \frac{\partial V_i}{\partial x_i} \quad (\rho = \mathrm{konstan})$$

Konveksi-Difusi

- Kuliah ini membahas sebaran massa
- Apabila air sungai mengalir, maka terjadi proses konveksi
- Persebaran polutan, dengan demikian, didorong oleh
  - beda konsentrasi (gradien) → difusi
  - aliran → konveksi

$$\frac{\partial c}{\partial t} + \underbrace{\vec{V} \cdot \overrightarrow{\mathrm{grad}}\, c}_{\text{konveksi}} = \underbrace{\mathrm{div}\left(\epsilon_m \overrightarrow{\mathrm{grad}}\, c\right)}_{\text{difusi}}$$

Konveksi-Difusi

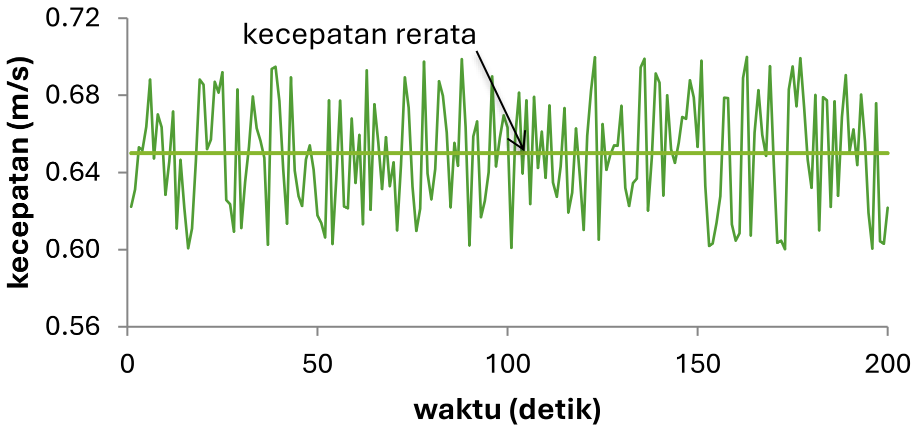
- Dituliskan dalam sistem koordinat cartesius

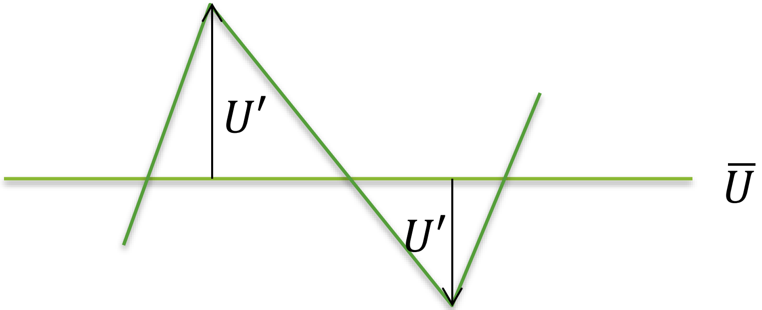
$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\frac{\partial}{\partial x}\epsilon_m\frac{\partial c}{\partial x}+\frac{\partial}{\partial y}\epsilon_m\frac{\partial c}{\partial y}+\frac{\partial}{\partial z}\epsilon_m\frac{\partial c}{\partial z}$$

$$\epsilon_m \text{ konstan} \Longrightarrow \frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\epsilon_m\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)$$

$$\underbrace{\frac{\partial c}{\partial t}=\epsilon_m\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)}_{\text{difusi murni}} \leftarrow \epsilon_m \text{ konstan, air diam tidak mengalir}$$

Konveksi-Difusi (Turbulen)





- Aliran di sungai hampir pasti adalah aliran turbulen
- Salah satu sifat aliran turbulen adalah bahwa kecepatan aliran berubah-ubah
- Konsentrasi polutan dengan demikian berubah-ubah pula

$$U = \overline{U} + U'$$

$$V = \overline{V} + V'$$

$$W = \overline{W} + W'$$

$$c = \bar{c} + c'$$

Konveksi-Difusi (Turbulen)

- Persamaan konveksi-difusi di aliran turbulen

$$\frac{\partial c}{\partial t} + \vec{V} \cdot \overrightarrow{\mathrm{grad}}\, c = \mathrm{div}\left[(\epsilon_m + \epsilon_t)\, \overrightarrow{\mathrm{grad}}\, c\right]$$

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial vc}{\partial y} + \frac{\partial wc}{\partial z} = \frac{\partial}{\partial x}\left[(\epsilon_m + \epsilon_{tx})\frac{\partial c}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\epsilon_m + \epsilon_{ty})\frac{\partial c}{\partial y}\right] + \frac{\partial}{\partial z}\left[(\epsilon_m + \epsilon_{tz})\frac{\partial c}{\partial z}\right]$$

- Koefisien difusi turbulen lazimnya jauh lebih besar daripada koefisien difusi molekuler, $\epsilon_t \gg \epsilon_m$
- Dalam bahasan konveksi-difusi turbulen, difusi molekuler diabaikan

Konveksi-Difusi (Turbulen)

- Persamaan konveksi-difusi di aliran turbulen
  - dituliskan dalam koordinat cartesius

$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\frac{\partial}{\partial x}\left(\epsilon_{tx}\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right)$$

$\epsilon_m + \epsilon_t \Longrightarrow \epsilon_t$ karena $\epsilon_t \gg \epsilon_m$

Persebaran Polutan

Difusi

Persamaan Difusi

- Persamaan difusi (air diam tidak mengalir)

$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\frac{\partial}{\partial x}\left[(\epsilon_m+\epsilon_{tx})\frac{\partial c}{\partial x}\right]+\frac{\partial}{\partial y}\left[(\epsilon_m+\epsilon_{ty})\frac{\partial c}{\partial y}\right]+\frac{\partial}{\partial z}\left[(\epsilon_m+\epsilon_{tz})\frac{\partial c}{\partial z}\right]$$

$$u=v=w=0 \quad u=0\Rightarrow\epsilon_{tx}=0 \quad v=0\Rightarrow\epsilon_{ty}=0 \quad w=0\Rightarrow\epsilon_{tz}=0$$

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\epsilon_m\frac{\partial c}{\partial x}+\frac{\partial}{\partial y}\epsilon_m\frac{\partial c}{\partial y}+\frac{\partial}{\partial z}\epsilon_m\frac{\partial c}{\partial z}$$

$$\epsilon_m\ \text{konstan}\Rightarrow\frac{\partial c}{\partial t}=\epsilon_m\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)$$

Persebaran Polutan

Difusi 1D

Difusi 1D

- Persamaan difusi satu dimensi

$$\frac{\partial c}{\partial t} = \epsilon_m \frac{\partial^2 c}{\partial x^2}$$

Difusi satu dimensi, arah x saja

- **Syarat batas** (*boundary conditions*) dan **syarat awal** (*initial conditions*) sbb.

$$c(\pm\infty, t) = 0$$

$$c(x, t = 0) = M_1 \delta(x)$$

$$M_0 = M_1 S$$

Difusi 1D

$$c(x, t = 0) = M_1 \delta(x)$$

$$M_0 = M_1 S$$

$$\int_{-\infty}^{+\infty} \delta(x) dx = 1$$

- M_1 adalah massa per satuan luas (kg/m^2) yang dimasukkan secara sekaligus dan tiba-tiba (*instantaneous source*)
- M_0 adalah seluruh massa yang dimasukkan di suatu titik secara tiba-tiba
- S adalah luas permukaan
- $\delta(x)$ adalah fungsi delta Dirac, bernilai sama dengan nol kecuali di $x = 0$

$$\int_{-\infty}^{+\infty} c(x, t) dx = \int_{-\infty}^{+\infty} c(x, 0) dx = M_1 \int_{-\infty}^{+\infty} \delta(x) dx = M_1$$

- Ingat bahwa jumlah massa M_0 harus konstan sepanjang waktu yang ditinjau

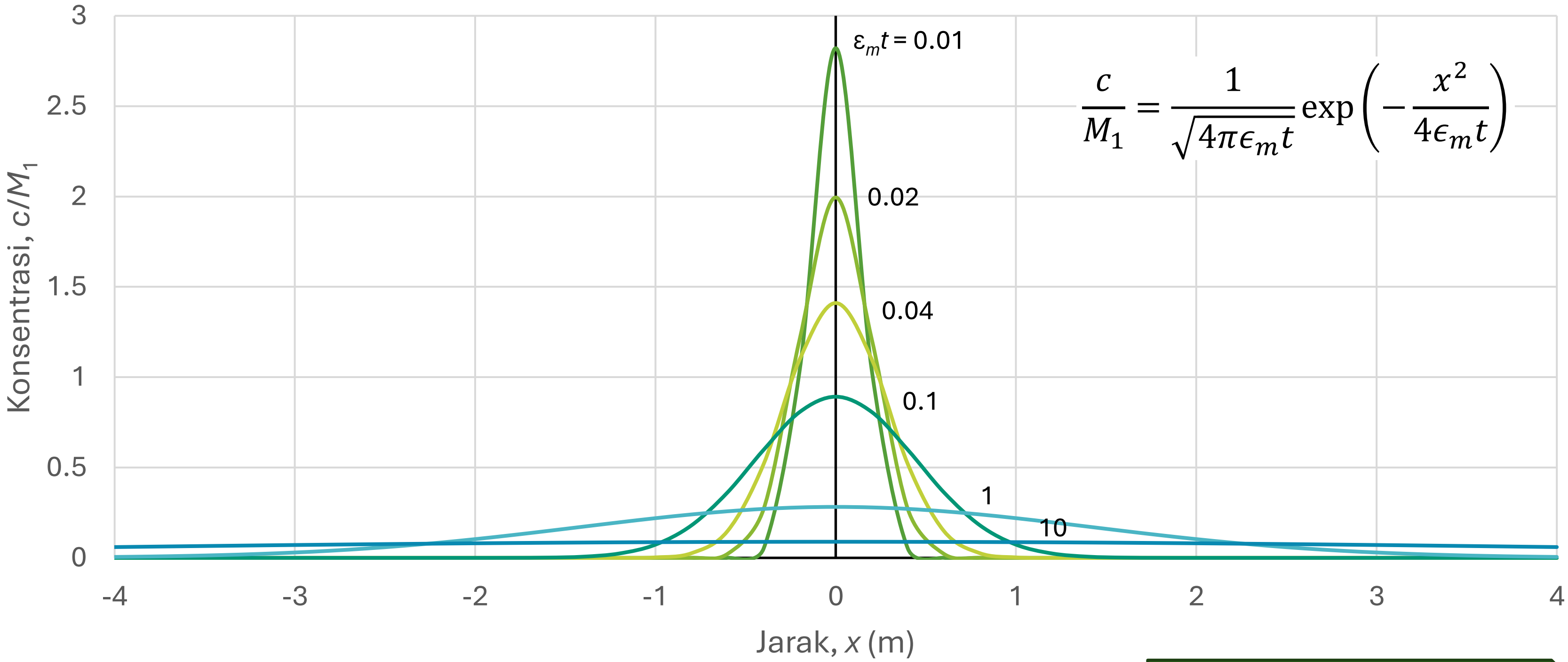
Difusi 1D

- Penyelesaian analitik persamaan difusi 1D tsb adalah

$$c(x,t) = \frac{M_1}{\sqrt{4\pi\epsilon_m t}} \exp\left(-\frac{x^2}{4\epsilon_m t}\right)$$

  - Penyelesaian tersebut menunjukkan difusi suatu massa M_0
    - yang dimasukkan secara tiba-tiba di titik $x = 0$;
    - kemudian menyebar ke arah sumbu $X-$ dan $X+$ menurut distribusi Gauss Normal dan simetris terhadap $x = 0$;
    - dan konsentrasi maksimum, yang berada di titik $x = 0$, berkurang seiring waktu.

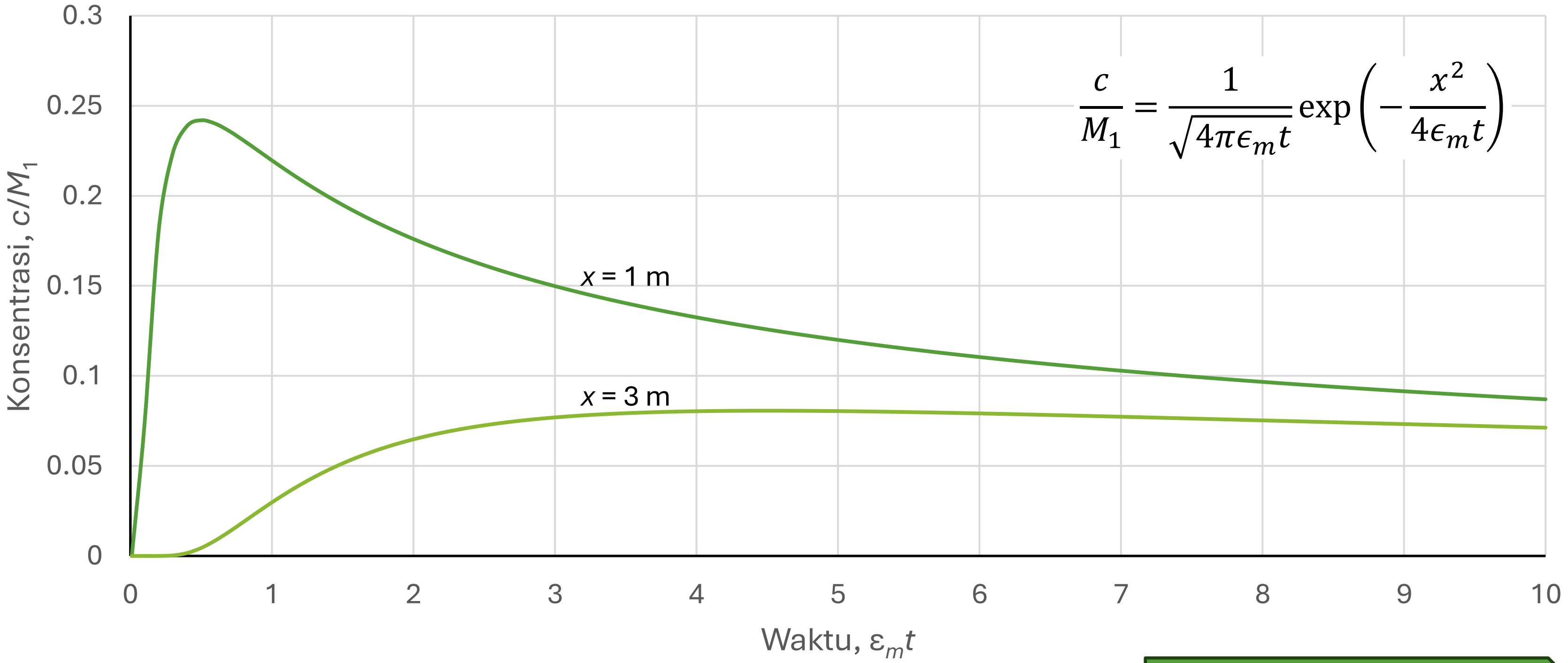
Difusi 1D



$$\frac{c}{M_1} = \frac{1}{\sqrt{4\pi\epsilon_m t}} \exp\left(-\frac{x^2}{4\epsilon_m t}\right)$$

Hitungan di *spreadsheet*

Difusi 1D



$$\frac{c}{M_1} = \frac{1}{\sqrt{4\pi\epsilon_m t}} \exp\left(-\frac{x^2}{4\epsilon_m t}\right)$$

Hitungan di *spreadsheet*

Difusi 1D

- Penyelesaian analitis persamaan difusi 1D tersebut dapat pula dituliskan sbb.

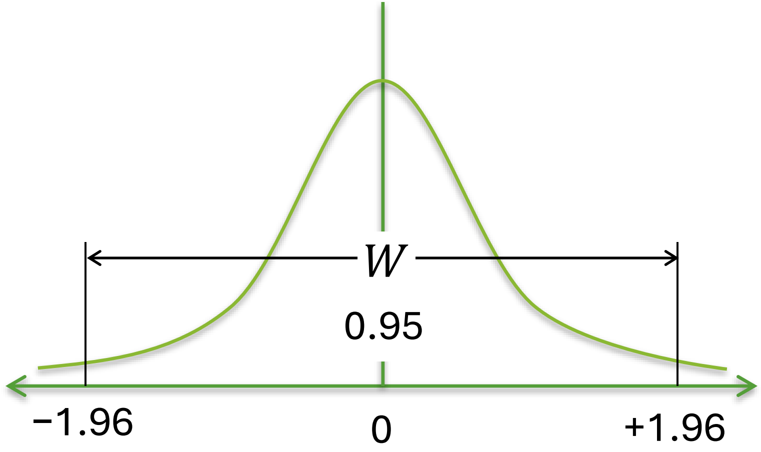
$$c(x,t) = \frac{M_1}{\sigma_X\sqrt{2\pi}}\exp\left(-\frac{x^2}{2{\sigma_X}^2}\right)$$

- Untuk suatu distribusi normal, varian distribusi adalah

$${\sigma_X}^2(t) = 2\epsilon_m t$$

- 95% luas daerah di bawah kurva pdf distribusi normal adalah

$$W = (2\times 1.96)\sigma_X \approx 4\sigma_X$$



Difusi 1D

- Koefisien difusi dapat dihitung sbb.

$$\epsilon_m = \frac{1}{2}\frac{d{\sigma_X}^2}{dt} = \frac{1}{2}\frac{{\sigma_X}^2(t_2) - {\sigma_X}^2(t_1)}{(t_2 - t_1)}$$

- Persamaan di atas dapat dipakai untuk mendapatkan koefisien difusi dengan cara mengukur simpangan baku konsentrasi polutan di titik x pada dua waktu yang berbeda, t_1 dan t_2

Difusi 1D di Medium Berbatas

- Penyelesaian analitis persamaan difusi 1D

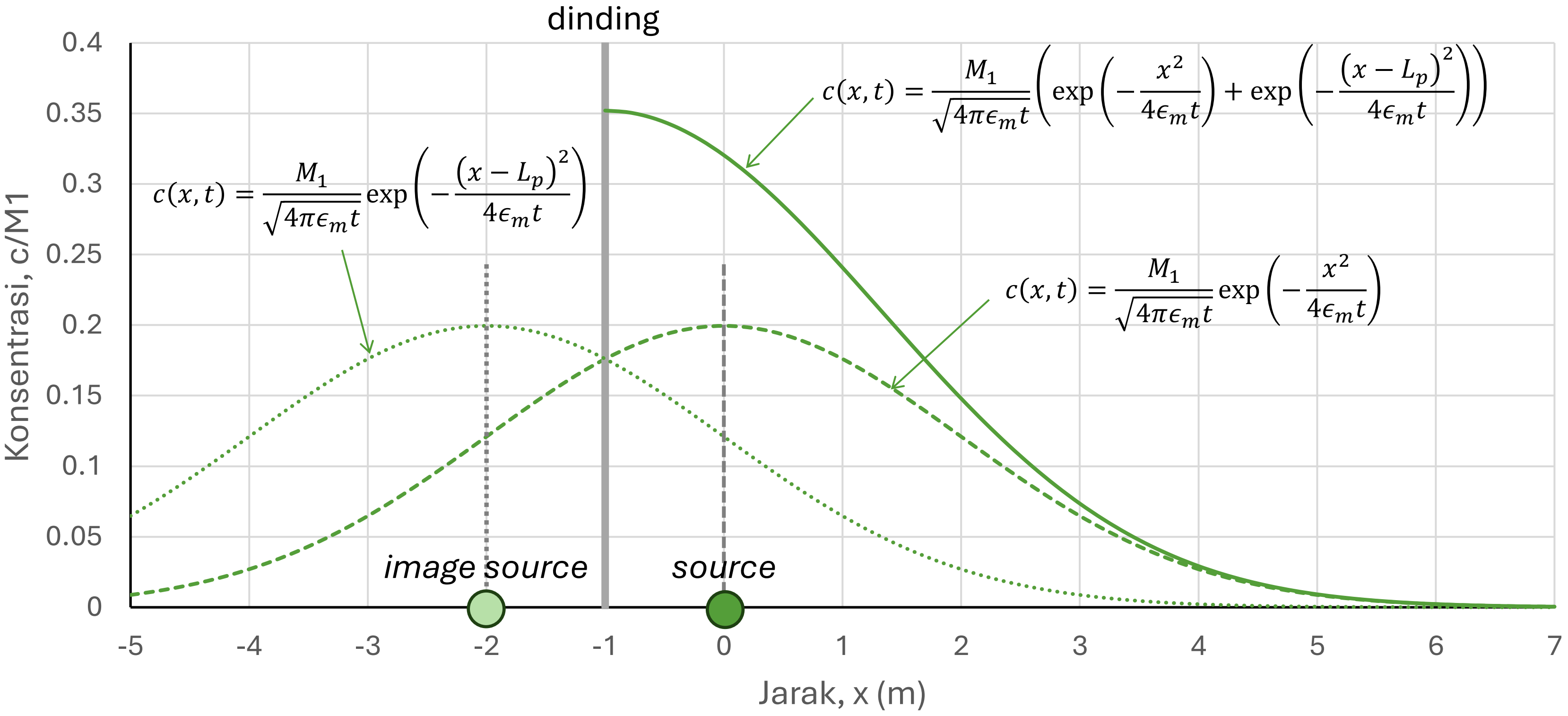
$$c(x,t) = \frac{M_1}{\sqrt{4\pi\epsilon_m t}} \exp\left(-\frac{x^2}{4\epsilon_m t}\right) \qquad c(x,t) = \frac{M_1}{\sigma_X\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_X{}^2}\right)$$

- Apabila medium memiliki batas dinding → pencerminan *source*

$$c(x,t) = \frac{M_1}{\sigma_X\sqrt{2\pi}} \left\{\exp\left(-\frac{x^2}{2\sigma_X{}^2}\right) + \exp\left(-\frac{\left(x-2L_p\right)^2}{2\sigma_X{}^2}\right)\right\}$$

$$c(x,t) = \frac{M_1}{\sqrt{4\pi\epsilon_m t}} \left\{\exp\left(-\frac{x^2}{4\epsilon_m t}\right) + \exp\left(-\frac{\left(x-2L_p\right)^2}{4\epsilon_m t}\right)\right\}$$

Difusi 1D di Medium Berbatas



Difusi 1D dari *Source* Menerus

- Persamaan difusi satu dimensi, massa M_0 dimasukkan secara menerus (kontinu) di titik $x = 0$

$$\frac{\partial c}{\partial t} = \epsilon_m \frac{\partial^2 c}{\partial x^2}$$

  - **Syarat batas** (*boundary conditions*) dan **syarat awal** (*initial conditions*) sbb.

$$c(x = 0, t \geq 0) = c_0 \qquad c(x > 0, t = 0) = 0$$

$$c(x = \pm\infty, t \geq 0) = 0 \qquad c(x < 0, t = 0) = 0$$

Difusi 1D dari *Source* Menerus

- Penyelesaian analitik persamaan difusi 1D tsb adalah

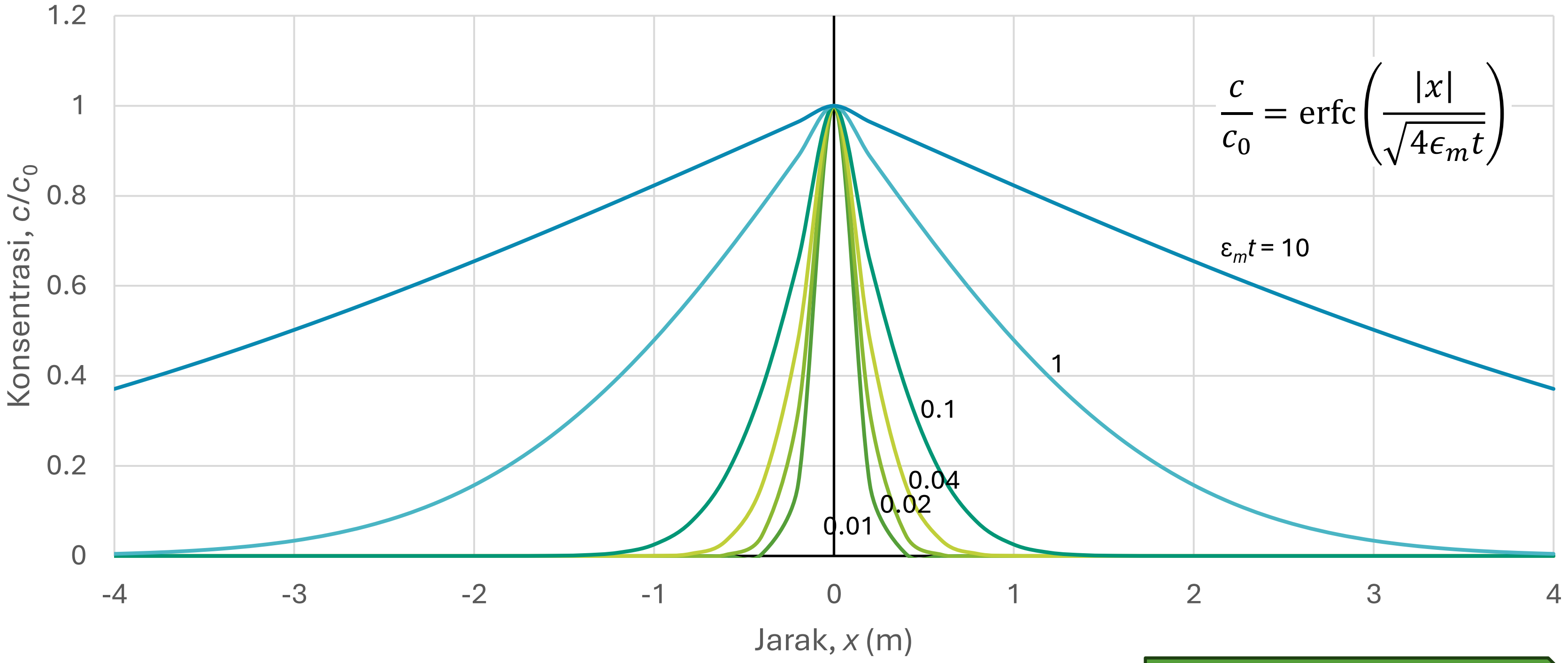
$$c(x,t) = c_0 \operatorname{erfc}\left(\frac{|x|}{\sqrt{4\epsilon_m t}}\right)$$

- *complementary error function*

$$\operatorname{erfc}(Y) = \frac{2}{\sqrt{\pi}} \int_Y^\infty e^{-\xi} d\xi$$

MS Excel
=ERFC(...)

Difusi 1D dari *Source* Menerus

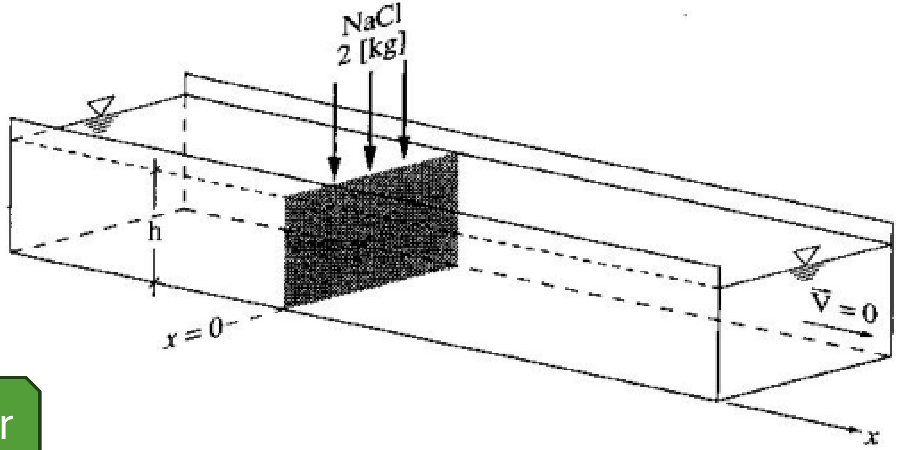


Hitungan di *spreadsheet*

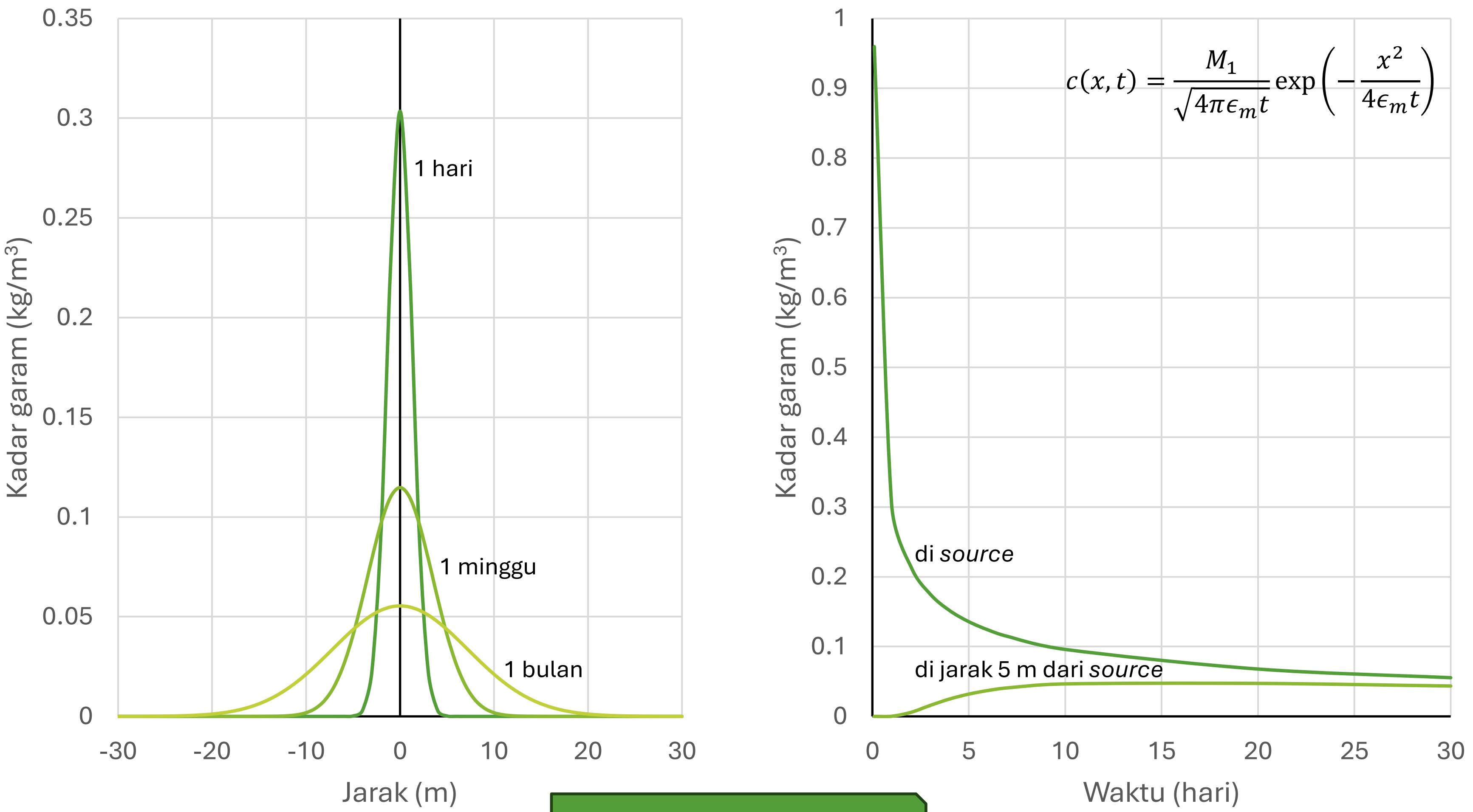
Ex. 8.A

A rectangular channel of width B = 2.0 [m], is filled with (stagnant) water, $\vec{V} = 0$, up to a depth of h = 1.0 [m]. A certain quantity, $M_0 = 2$ [kg], of common salt (NaCl), released into the water at the station $x = 0.0$ [m], spreads immediately over the entire cross section of the channel.

Determine the evolution in time : *i)* of the longitudinal spreading of the salt and *ii)* of its maximum concentration. Plot afterwards : *iii)* the concentration distribution for the time $t_R = 10^7$[s].



Difusi molekular



$$c(x,t) = \frac{M_1}{\sqrt{4\pi\epsilon_m t}} \exp\left(-\frac{x^2}{4\epsilon_m t}\right)$$

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Persebaran Polutan

Difusi 2D

Difusi 2D

- Persamaan difusi dua dimensi

$$\frac{\partial c}{\partial t} = \epsilon_m \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)$$

Difusi dua dimensi, arah x dan y (bidang z)

- **Syarat batas** (*boundary conditions*) dan **syarat awal** (*initial conditions*) sbb.

$$c(\pm\infty, \pm\infty, t) = 0$$

$$c(x, y, t = 0) = M_2 \delta(x, y)$$

Difusi 2D

- Penyelesaian analitik persamaan difusi 2D tsb adalah

$$c(x,y,t) = \frac{M_x}{\sigma_X\sqrt{2\pi}}\exp\left(-\frac{x^2}{2{\sigma_X}^2}\right) + \frac{M_y}{\sigma_Y\sqrt{2\pi}}\exp\left(-\frac{y^2}{2{\sigma_Y}^2}\right)$$

- Apabila medium homogen

$$\sigma_X = \sigma_Y = \sigma$$

$$c(x,y,t) = \frac{M_2}{\left(\sigma\sqrt{2\pi}\right)^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$\sigma^2(t) = 2\epsilon_m t$$

$$M_2 = M_0/L$$

Persebaran Polutan

Difusi 3D

Difusi 3D

- Persamaan difusi dua dimensi

$$\frac{\partial c}{\partial t} = \epsilon_m \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right)$$

Difusi tiga dimensi, arah x, y, dan z

- **Syarat batas** (*boundary conditions*) dan **syarat awal** (*initial conditions*) sbb.

$$c(\pm\infty, \pm\infty, \pm\infty, t) = 0$$

$$c(x, y, z, t = 0) = M_3 \delta(x, y, z)$$

Difusi 3D

- Penyelesaian analitik persamaan difusi 3D tsb adalah

$$c(x,y,z,t) = \frac{M_3}{\left(\sigma\sqrt{2\pi}\right)^3}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$r^2 = x^2 + y^2 + z^2$$

$$M_3 = M_0$$

Persebaran Polutan

Konveksi-Difusi

Konveksi-Difusi (Turbulen)

- Persamaan konveksi-difusi dalam aliran turbulen

$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\frac{\partial}{\partial x}\left(\epsilon_{tx}\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right)$$

- Koefisien difusi merupakan besaran tensorial
  - Koefisien difusi vertikal, ϵ_{tz}
  - Koefisien difusi transversal, ϵ_{ty}
  - Koefisien difusi longitudinal, ϵ_{tx}

$$\vec{\vec{\epsilon}}_t=\left(\epsilon_{tx},\epsilon_{ty},\epsilon_{tz}\right)$$

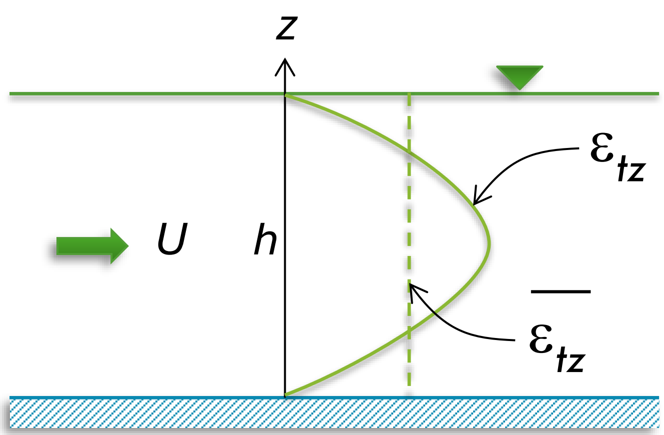
Konveksi-Difusi (Turbulen)

- Koefisien difusi vertikal

$$\epsilon_{tz} = \kappa u_* \frac{z}{h}(h - z)$$

- Koefisien difusi vertikal rerata kedalaman aliran

$$\overline{\epsilon_{tz}} = \frac{1}{h}\int_0^h \kappa u_* \frac{z}{h}(h - z)dz$$

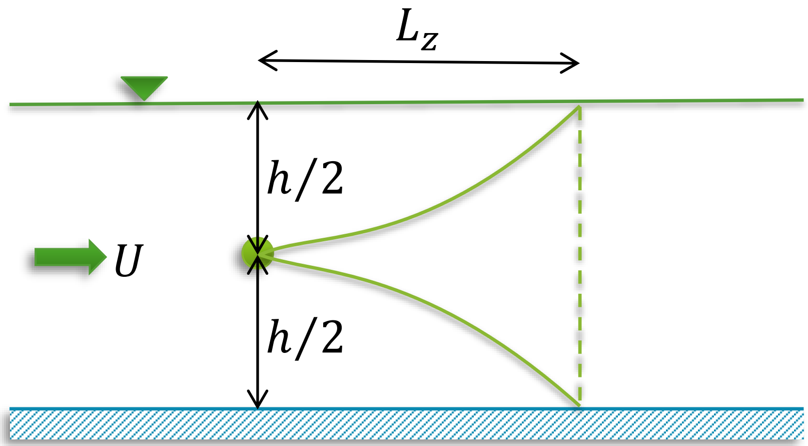


$$\overline{\epsilon_{tz}} = 0.067(hu_*)$$

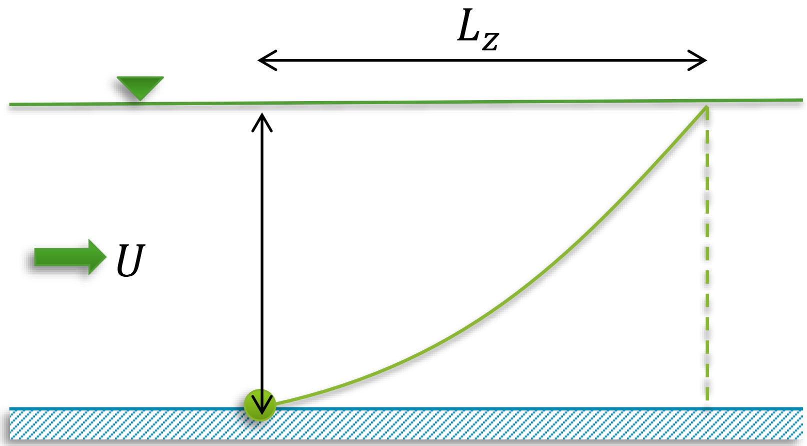
kecepatan geser

kedalaman aliran

Konveksi-Difusi (Turbulen)



$\xi_z = 0.1$



$\xi_z = 0.4$

Jarak $L_z = \xi_z U \dfrac{h^2}{\epsilon_{tz}}$ ditempuh dalam waktu $t_z = \xi_z \dfrac{h^2}{\epsilon_{tz}}$

U kecepatan rerata
kedalaman aliran

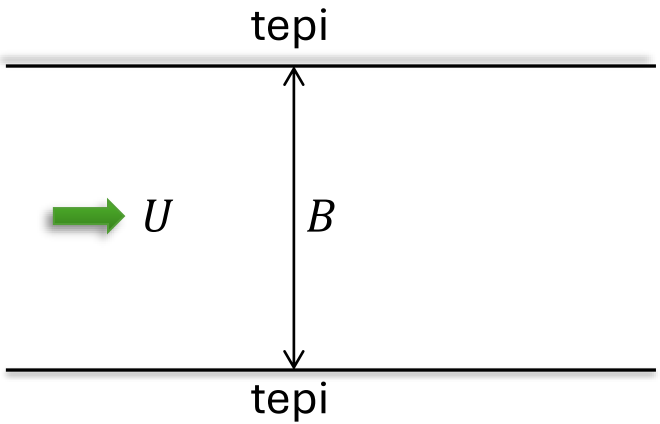
Konveksi-Difusi (Turbulen)

- Koefisien difusi transversal
  - di *flume*

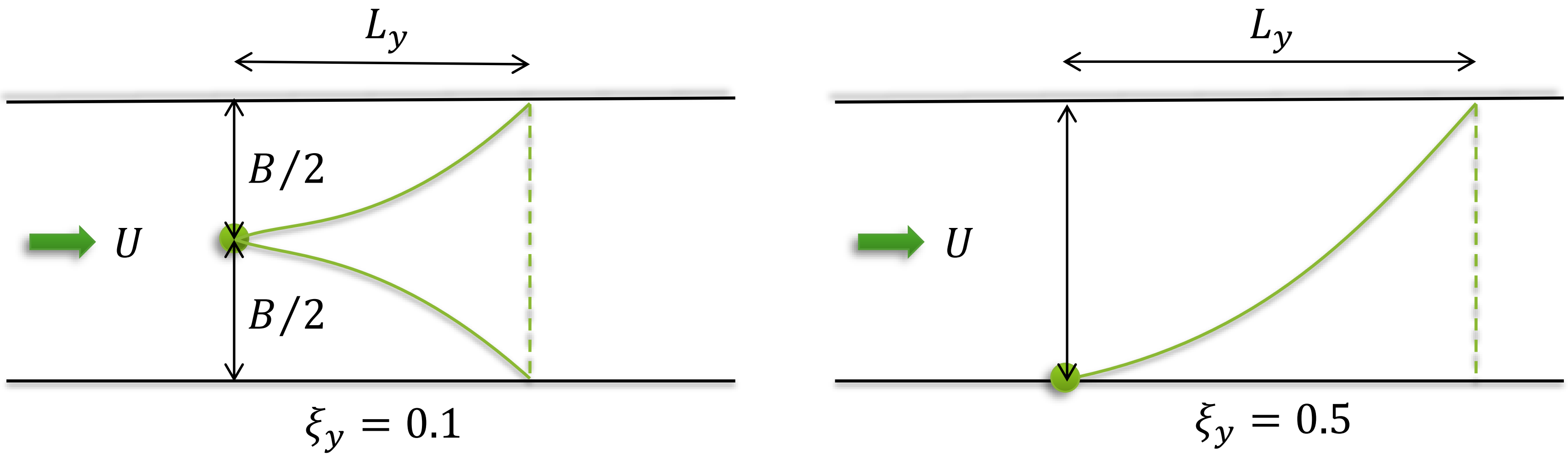
    $$\overline{\epsilon_{ty}} = 0.15(hu_*)$$

  - di sungai

    $$\overline{\epsilon_{ty}} = 0.6(hu_*)$$



Konveksi-Difusi (Turbulen)



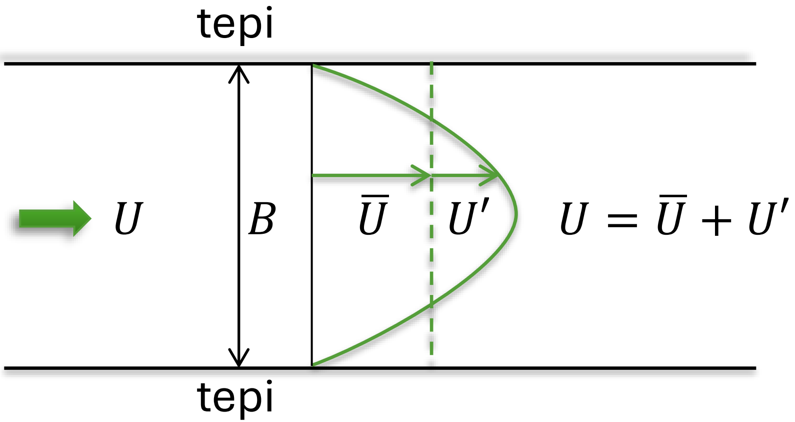
Jarak $L_y = \xi_y U \dfrac{B^2}{\epsilon_{ty}}$ ditempuh dalam waktu $t_y = \xi_y \dfrac{B^2}{\epsilon_{ty}}$

U kecepatan rerata
kedalaman aliran

Konveksi-Difusi (Turbulen)

- Koefisien difusi longitudinal, searah aliran

$$\overline{\epsilon_{tx}} = 0.23(hu_*)$$



  - Difusi longitudinal (searah aliran) yang ditimbulkan oleh turbulensi aliran umumnya diabaikan karena pengaruh **dispersi** lebih dominan.
  - Parameter dispersi adalah koefisien dispersi K_x.
  - Dispersi terjadi karena adanya variasi besaran kecepatan aliran (distribusi kecepatan) → beda antara kecepatan rerata dan kecepatan di suatu titik.

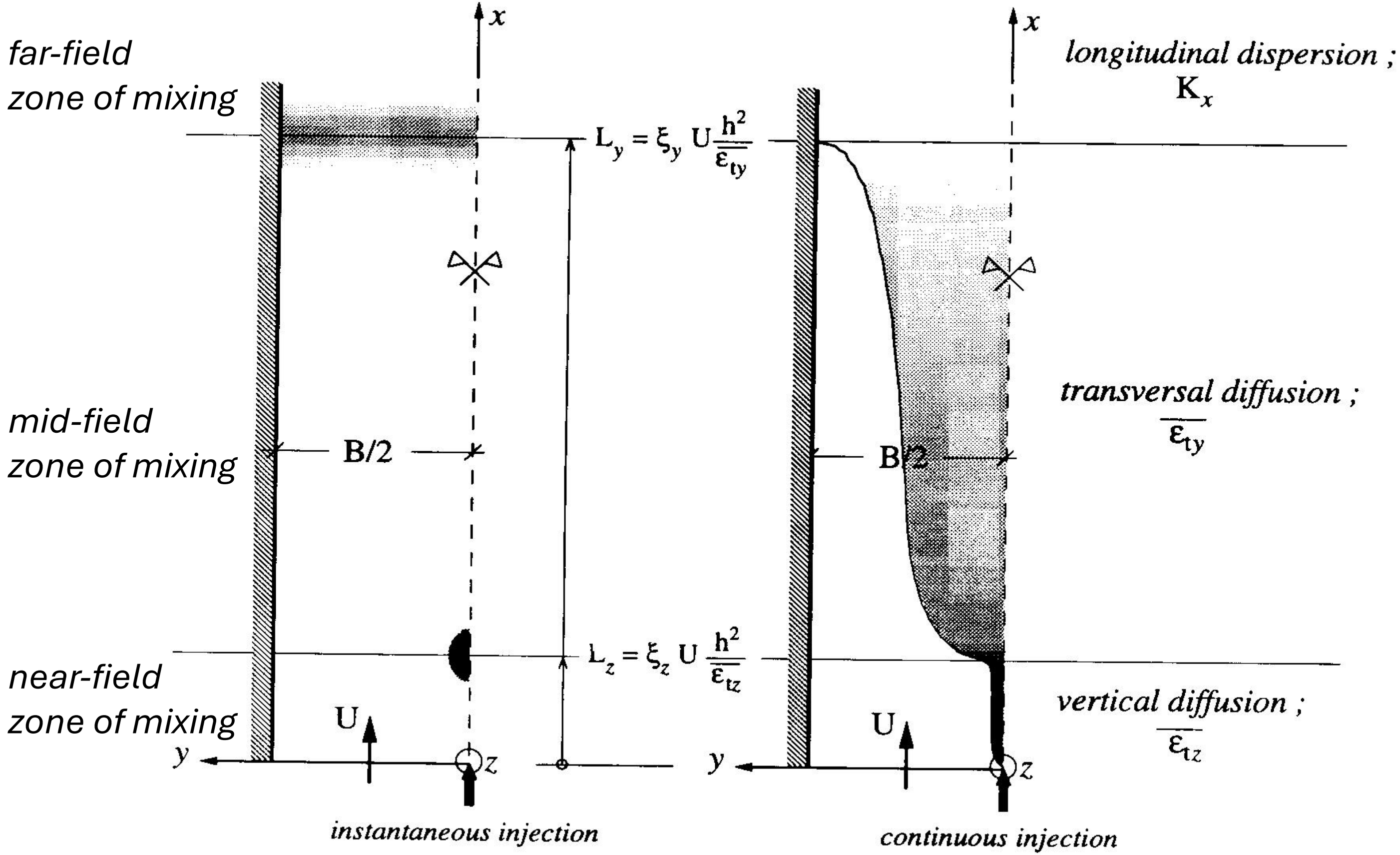


Fig. 8.8 Scheme of the convection-diffusion after an instantaneous and continuous injection; the distances, L_z and L_y, of total mixing are shown.

Persebaran Polutan

Konveksi-Difusi Vertikal

Konveksi-Difusi Vertikal

- Persamaan konveksi-difusi dalam aliran turbulen (difusi molekular diabaikan)

$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\frac{\partial}{\partial x}\left(\epsilon_{tx}\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right)$$

- Apabila
  - aliran hanya satu arah, $u \neq 0,\, v = w = 0 \Rightarrow \vec{V}(u, 0,0) = U$
  - sumber polutan kontinu
  - transpor polutan dianggap permanen
  - difusi longitudinal diabaikan
  - difusi transversal telah langsung dicapai, sumber polutan merata dari tebing kiri ke kanan sungai

Konveksi-Difusi Vertikal

$$\cancel{\frac{\partial c}{\partial t}} + \frac{\partial uc}{\partial x} + \cancel{\frac{\partial vc}{\partial y}} + \cancel{\frac{\partial wc}{\partial z}} = \frac{\partial}{\partial x}\left(\epsilon_{tx}\cancel{\frac{\partial c}{\partial x}}\right) + \frac{\partial}{\partial y}\left(\epsilon_{ty}\cancel{\frac{\partial c}{\partial y}}\right) + \frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right)$$

transpor permanen

$v = w = 0$

difusi longitudinal diabaikan

difusi transversal telah dicapai

$$\frac{\partial uc}{\partial x} = \frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right) \Rightarrow U\frac{\partial c}{\partial x} = \epsilon_{tz}\frac{\partial^2 c}{\partial z^2}$$

ϵ_{tz} koefisien difusi konstan

U kecepatan aliran rerata kedalaman (*depth-averaged velocity*)

Konveksi-Difusi Vertikal

$$\underbrace{\frac{\partial uc}{\partial x} = \frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right) \Longrightarrow U\frac{\partial c}{\partial x} = \epsilon_{tz}\frac{\partial^2 c}{\partial z^2}}$$

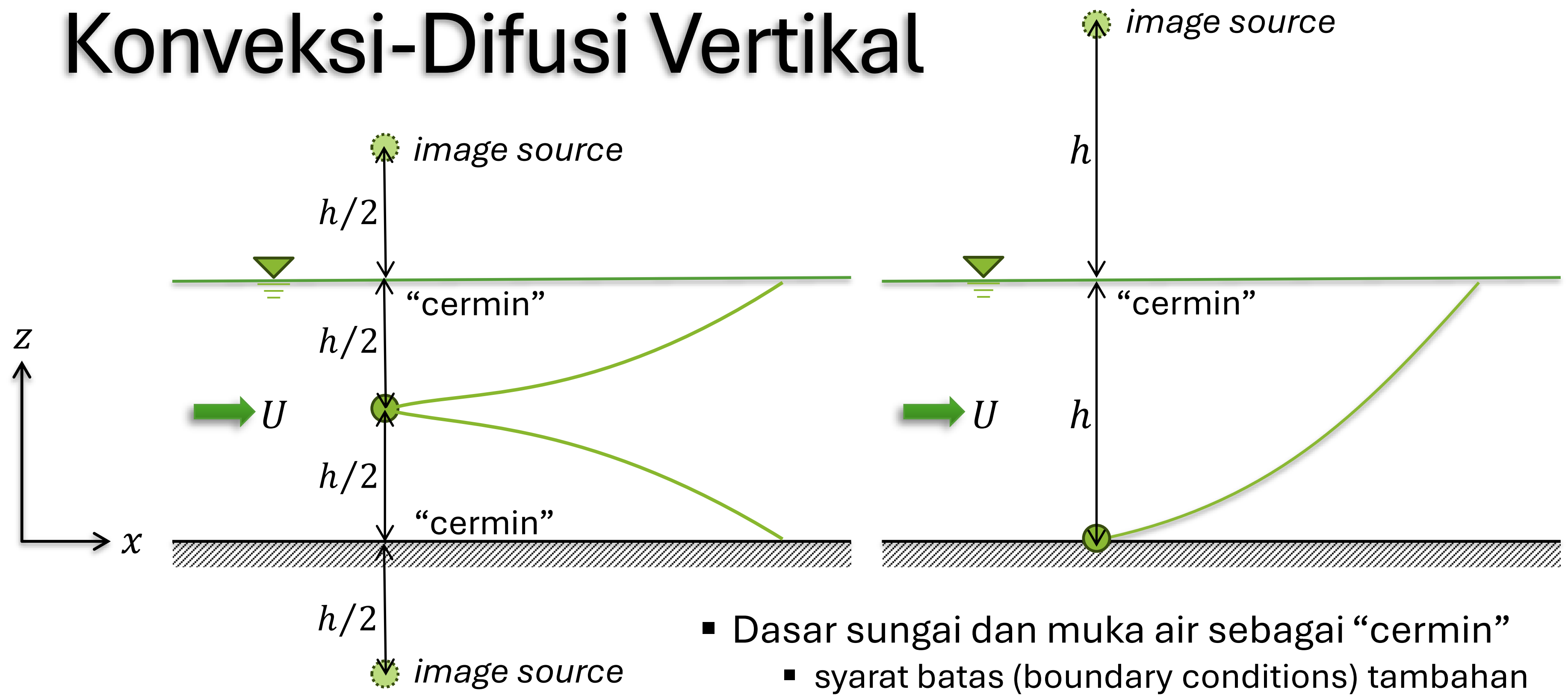
Penyelesaian analitik persamaan di atas apabila medium sangat luas adalah

$$G_u = c_u Q_u$$

$$c_u(x,z) = \frac{G_u}{B\sqrt{4\pi\epsilon_{tz}xU}}\exp\left(-\frac{z^2\,U}{4\epsilon_{tz}x}\right)$$

G_u adalah debit polutan di *source* yang diinjeksikan merata ke seluruh lebar sungai/saluran secara kontinu

Konveksi-Difusi Vertikal



- Dasar sungai dan muka air sebagai "cermin"
  - syarat batas (boundary conditions) tambahan

$$\left.\frac{\partial c}{\partial z}\right|_{z=0} = 0 \qquad \left.\frac{\partial c}{\partial z}\right|_{z=h} = 0$$

Konveksi-Difusi Vertikal

$$c_u(x,z) = \frac{G_u}{B\sqrt{4\pi\epsilon_{tz}xU}}\exp\left(-\frac{z^2\,U}{4\epsilon_{tz}x}\right)$$

konsentrasi polutan di bidang tak berbatas

Konsentrasi polutan di suatu titik (lokasi) berasal dari *source* original dan *source* imajiner karena pencerminan oleh dasar sungai dan muka air

konsentrasi polutan di bidang berbatas

$$c(x,z) = c_u(x,z+z_0) + c_u(x,z-z_0)$$

$$c(x,z) = c_u(x,z+z_0) + c_u(x,z-z_0) + \underbrace{\sum_{n=1}^{N} c_u(x,2nh \pm z \pm z_0)}$$

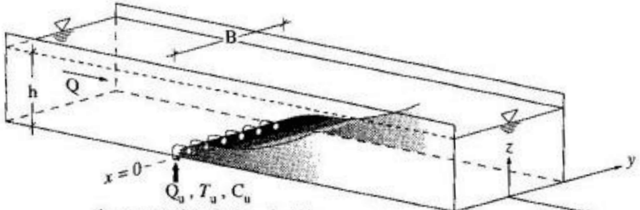
Pencerminan dapat terjadi berulang kali → *image source* dicerminkan kembali oleh dasar sungai dan muka air

z_0 adalah posisi *source* terhadap bidang referensi (dasar sungai)

Konveksi-difusi vertikal

Ex. 8.B

In a rectangular channel the flow is controlled and has a constant discharge of $Q = 103.7$ [m³/s]. The following hydraulic parameters are measured : $U = 1.05$ [m/s], $A = 98.8$ [m²], $h = 1.09$ [m], $B = 90.6$ [m], $u_* = 0.07$ [m/s] and $S_f = 0.0005$ [–]; the uniform flow in the channel is subcritical and turbulent. The water is at a temperature of $T = 10$ [°C] and has a salt content of $C = 1$ [kg/m³].



linear source for unit channel width; injection at the bottom

This channel receives wastewater from a nearby treatment plant. A continuous discharge, $Q_u = 0.5$ [m³/s] , of wastewater is released uniformly over the entire width, B, of the channel by means of a multipoint-diffuser system installed at its bottom. The wastewater has a temperature of $T_u = 20$ [°C] and a salt content of $C_u = 30$ [kg/m³].

Study the *vertical* diffusion of the injected wastewater.

i) Derive the hydraulic relationships for the vertical diffusion.

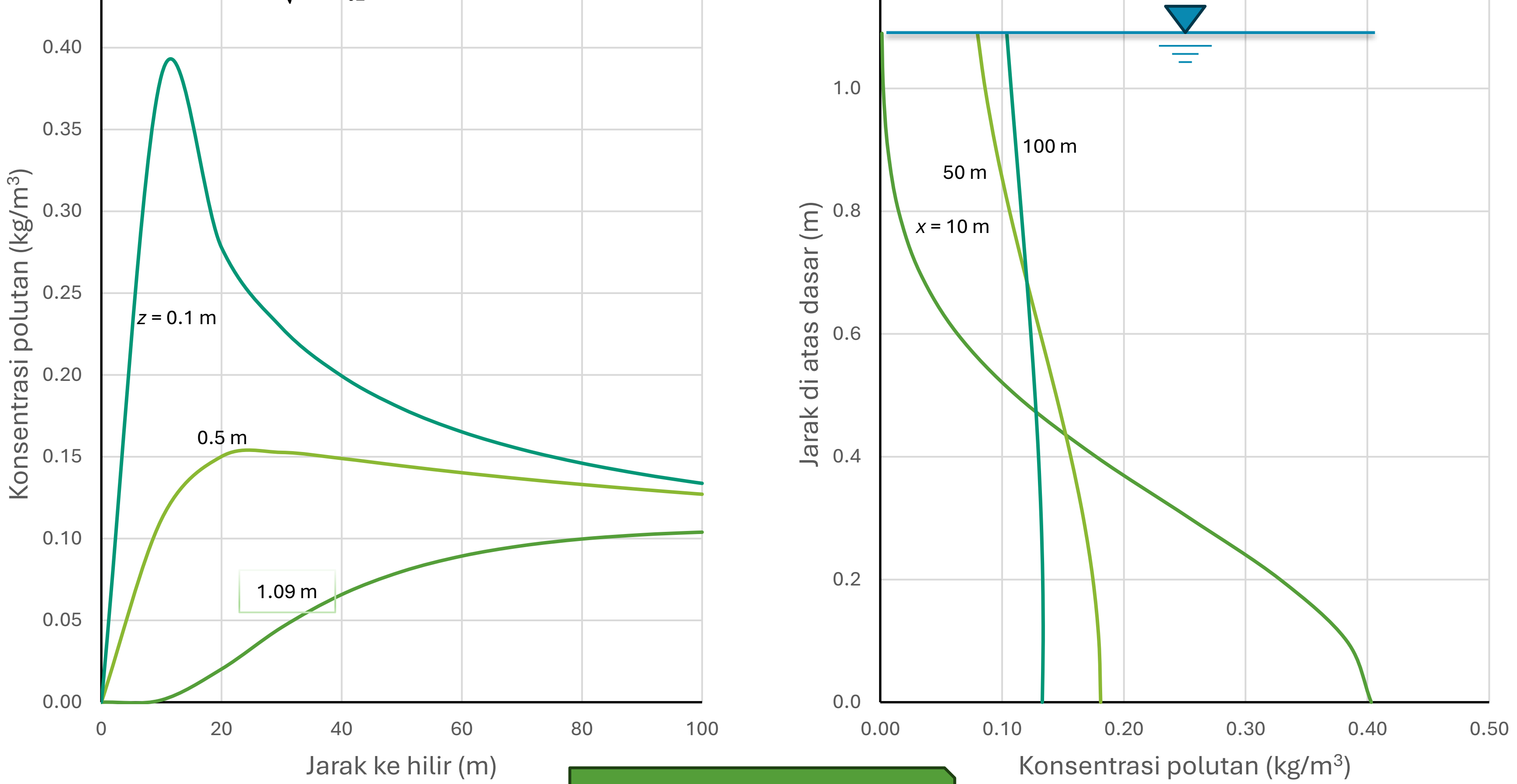
ii) Determine the distance required for the vertical diffusion to be completed over the entire flow depth, h.

iii) Find the distance from the source of the point at which a concentration, being equal to the half of the depth-averaged final concentration, $c = C_M/2$, appears at the water surface, h.

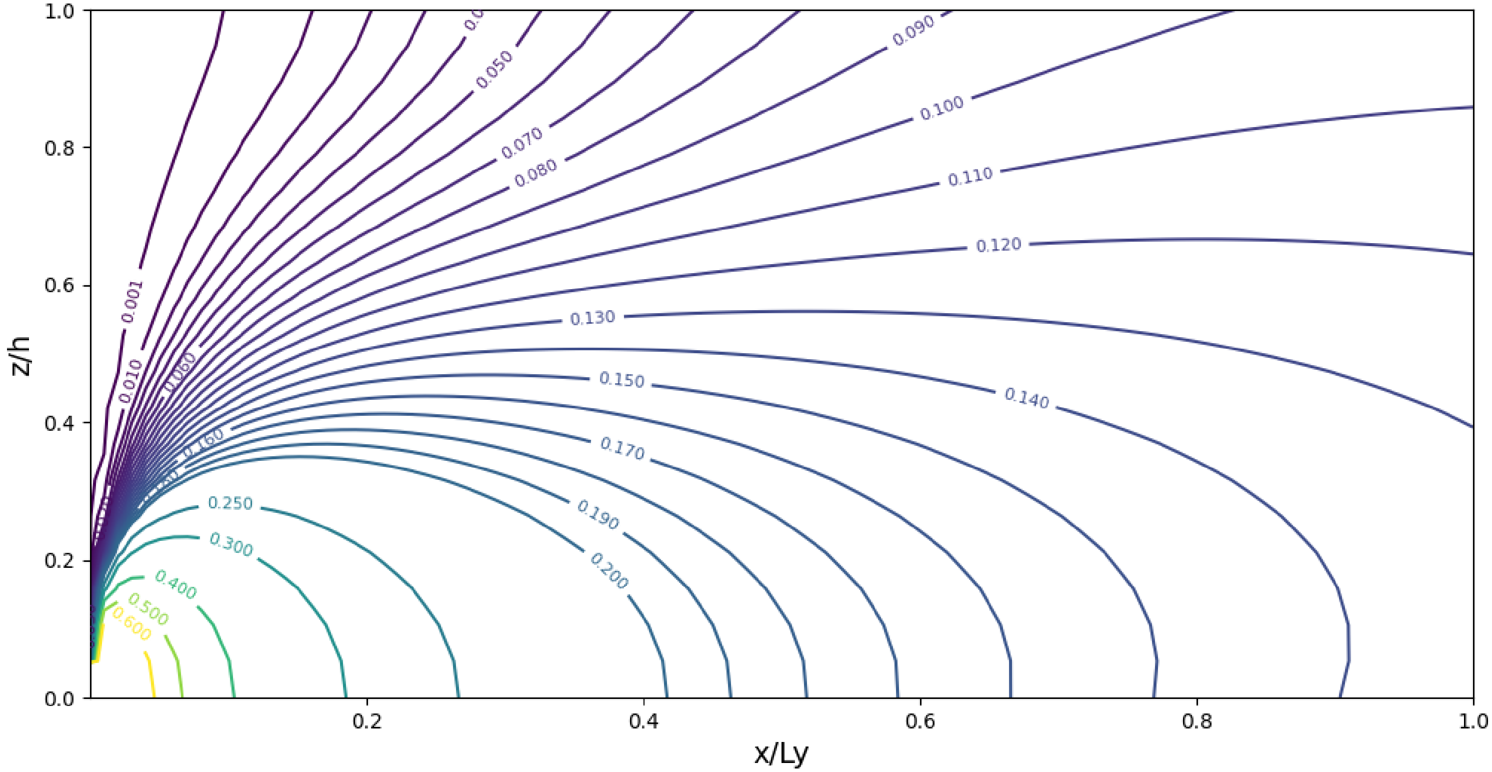
iv) Calculate and plot the evolution of the salt content and of the temperature as a function of the distance.

v) Propose a method for measuring the vertical diffusivity, ε_{tz} .

$$c(x,z)=\frac{G_u}{B\sqrt{4\pi\epsilon_{tz}xU}}\left\{\exp\left(-\frac{z^2\,U}{4\epsilon_{tz}x}\right)+\exp\left(-\frac{(z-2h)^2\,U}{4\epsilon_{tz}x}\right)+\exp\left(-\frac{z^2\,U}{4\epsilon_{tz}x}\right)\right\}$$



Hitungan di *spreadsheet*



Konsentrasi polutan dalam satuan kg/m^3
(program kontur dituliskan dalam bahasa pemrograman Python)

Persebaran Polutan

Konveksi-Difusi Transversal

Konveksi-Difusi Transversal

- Persamaan konveksi-difusi dalam aliran turbulen

$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\frac{\partial}{\partial x}\left(\epsilon_{tx}\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right)$$

- Apabila
  - aliran hanya satu arah, $u \neq 0, v = w = 0$
  - sumber polutan kontinu dan transpor polutan dianggap permanen
  - difusi longitudinal diabaikan
  - difusi vertikal telah dicapai, polutan telah menyebar di seluruh kedalaman aliran

Konveksi-Difusi Transversal

$$\cancel{\frac{\partial c}{\partial t}} + \frac{\partial uc}{\partial x} + \cancel{\frac{\partial vc}{\partial y}} + \cancel{\frac{\partial wc}{\partial z}} = \frac{\partial}{\partial x}\left(\epsilon_{tx}\cancel{\frac{\partial c}{\partial x}}\right) + \frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\epsilon_{tz}\cancel{\frac{\partial c}{\partial z}}\right)$$

transpor permanen

$v = w = 0$

difusi longitudinal diabaikan

difusi vertikal telah dicapai

$$\frac{\partial uc}{\partial x} = \frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right) \Rightarrow U\frac{\partial C}{\partial x} = \epsilon_{ty}\frac{\partial^2 C}{\partial y^2}$$

karena polutan telah menyebar di seluruh kedalaman aliran, maka tinjauan dilakukan untuk nilai rerata kedalaman

U kecepatan aliran rerata kedalaman (*depth-averaged velocity*)

C konsentrasi polutan rerata kedalaman (*depth-averaged concentration*)

Konveksi-Difusi Transversal

- Penyelesaian analitis persamaan konveksi-difusi transversal di sungai yang lebar

$$C_u(x,y) = \frac{G_0}{h\sqrt{4\pi\epsilon_{ty}xU}} \exp\left(-\frac{y^2U}{4\epsilon_{ty}x}\right)$$

$G_0 = M_0/t$ (kg/s)
debit polutan, merata di seluruh kedalaman aliran

- Penyelesaian analitis persamaan konveksi-difusi transversal di sungai berbatas

$$C(x,y) = C_u(x,y+y_0) + \sum_{n=1}^{N} C_u(x, 2nB \pm y \pm y_0)$$

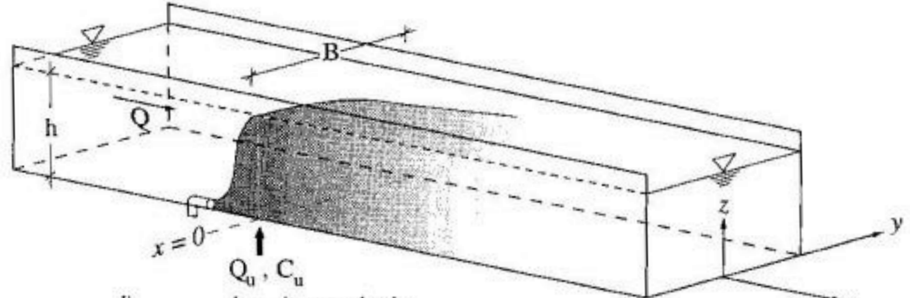
y_0 lokasi sumber polutan

Konveksi-difusi transversal

Ex. 8.C

A channel designed for a constant discharge of $Q = 103.7$ [m³/s], has the following hydraulic parameters : $U = 1.05$ [m/s], $h = 1.09$ [m], $B = 90.6$ [m], $u_* = 0.07$ [m/s] and $S_f = 0.0005$ [–]. The flow in the channel is uniform, subcritical and turbulent.

Wastewater with a salt content of $C_u = 30$ [kg/m³] is discharged into this channel at a rate of $Q_u = 0.5$ [m³/s].

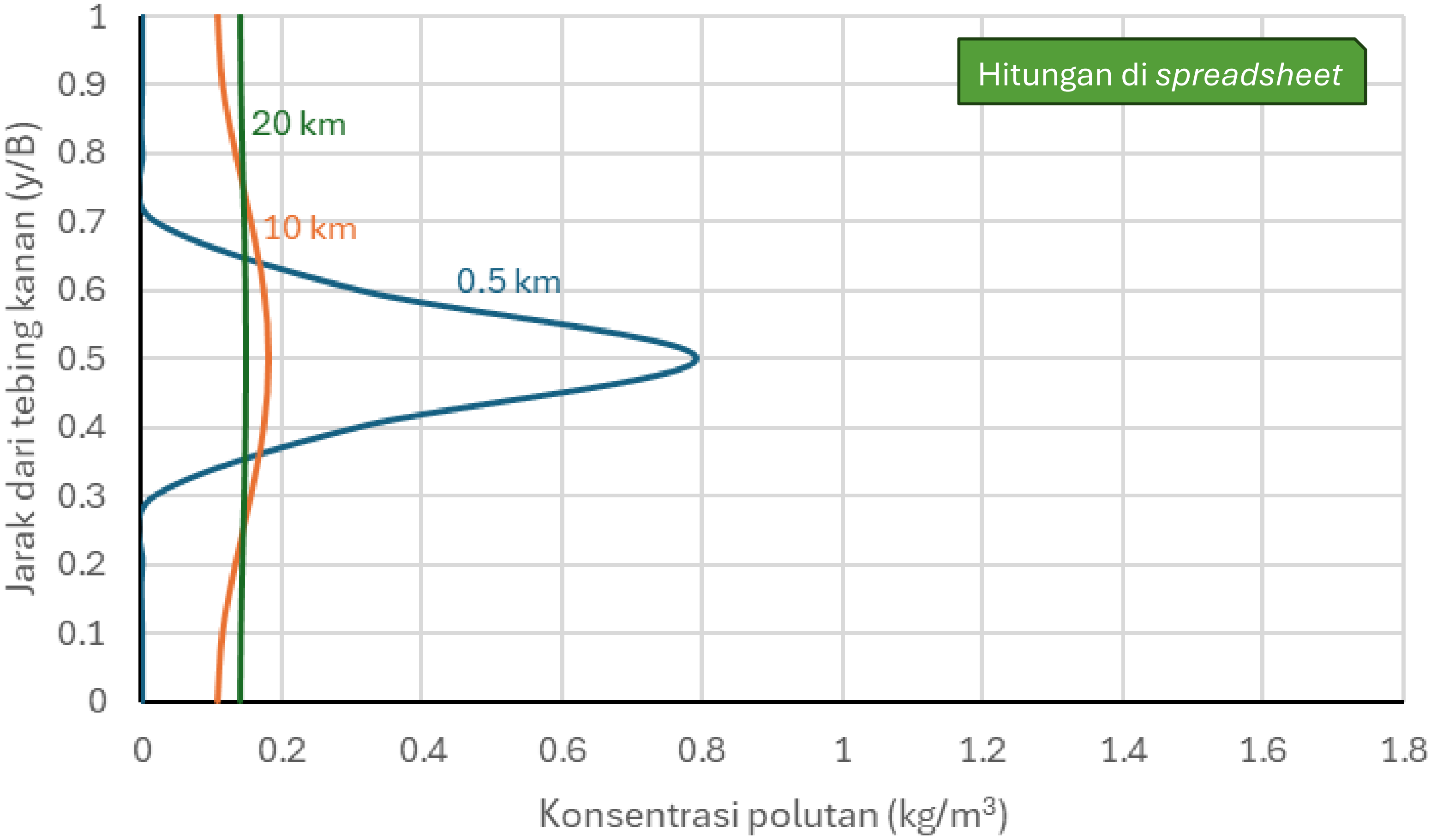


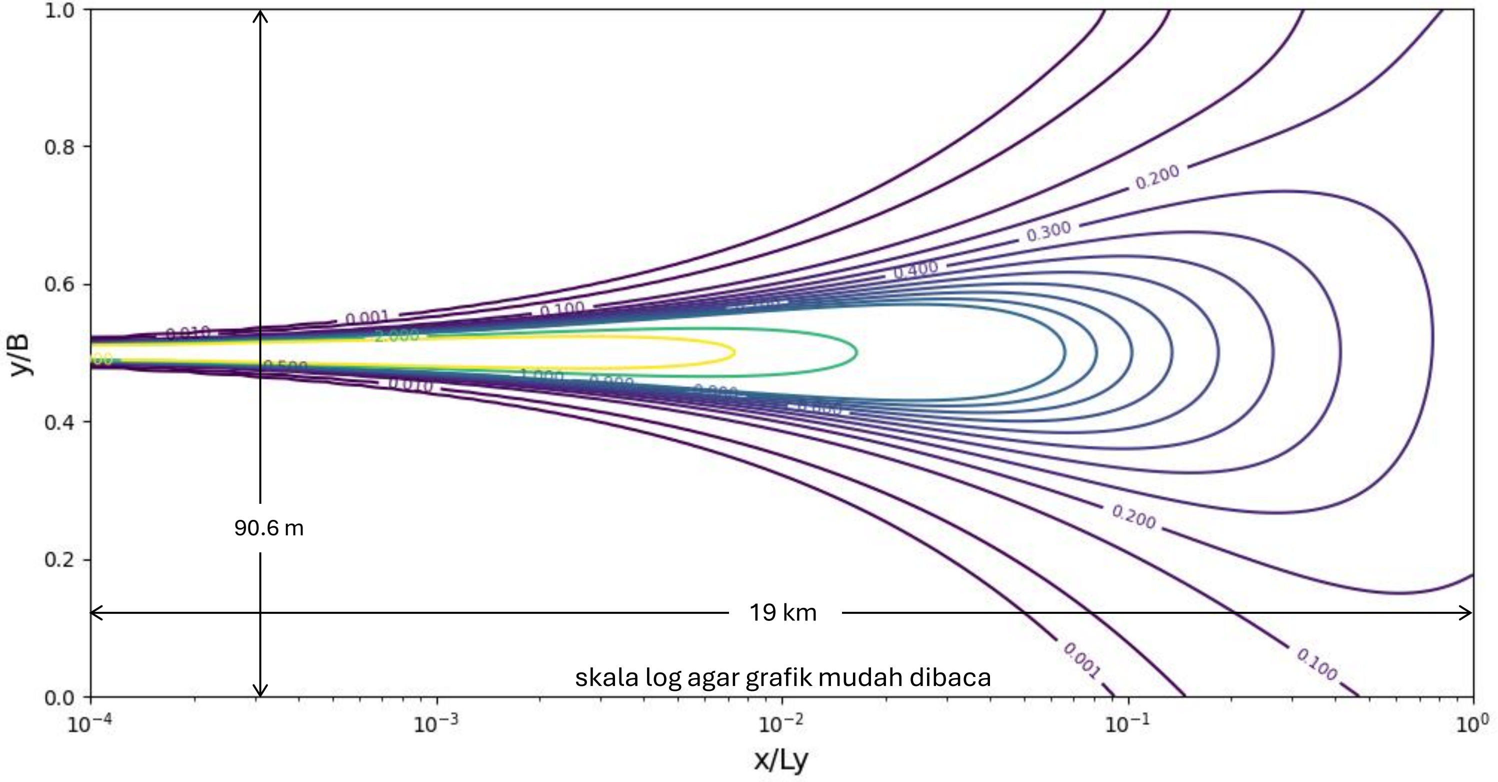
linear source by unit water depth;
injection on the side wall

Study the *transversal* diffusion for the following scenarios where the wastewater is injected either at the center of the channel or at one of the side walls. It shall be admitted that the vertical diffusion takes place almost instantly (see point 8.4.1,7°).

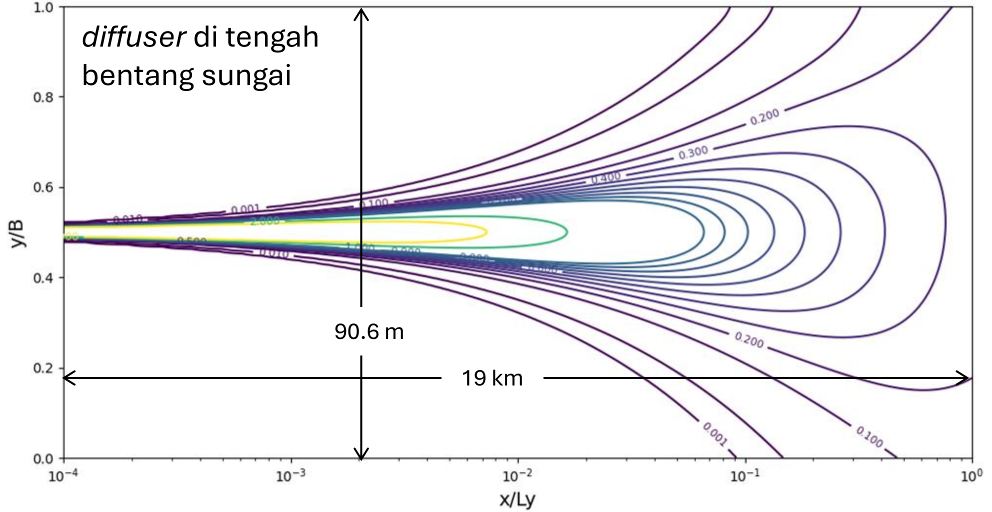
i) Determine the distance necessary to achieve a complete transversal diffusion over the entire width of the channel for both of the injection scenarios.

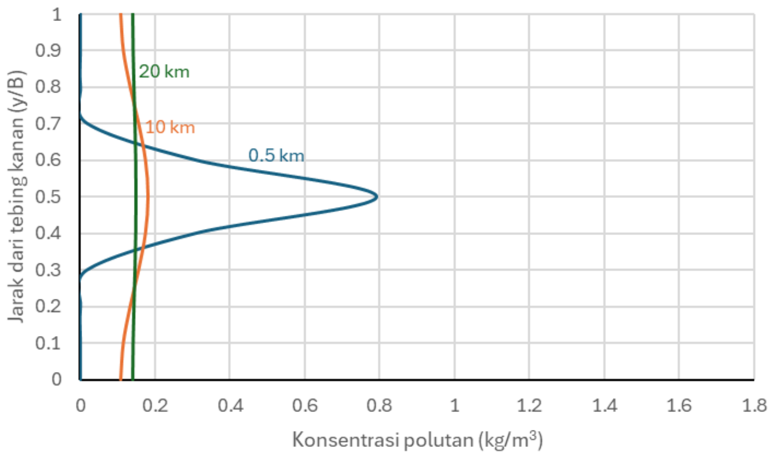
ii) Calculate and plot the transversal wastewater concentration profile at the stations $L_1 = 0.5$ [km] and $L_2 = 20$ [km].

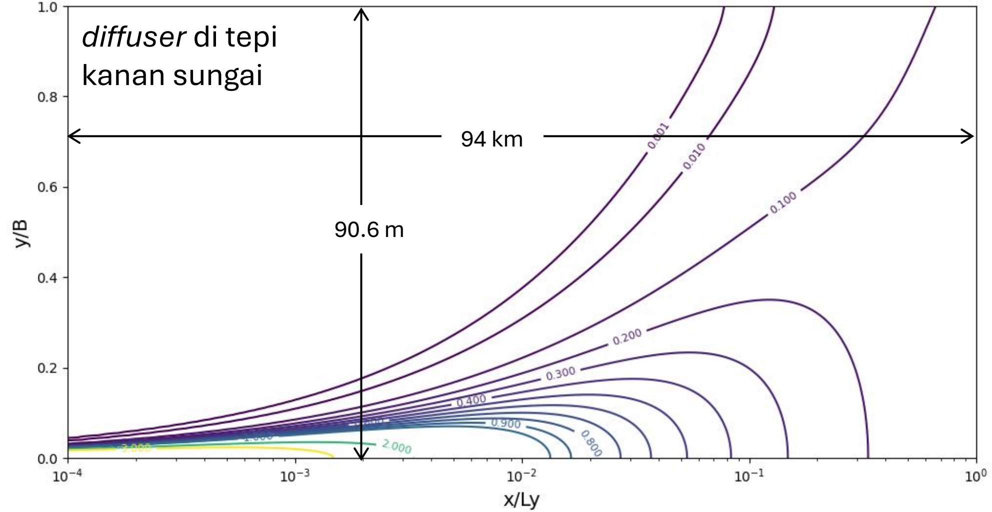

Hitungan di *spreadsheet*
20 km
10 km
0.5 km
Jarak dari tebing kanan (y/B)
Konsentrasi polutan (kg/m3)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

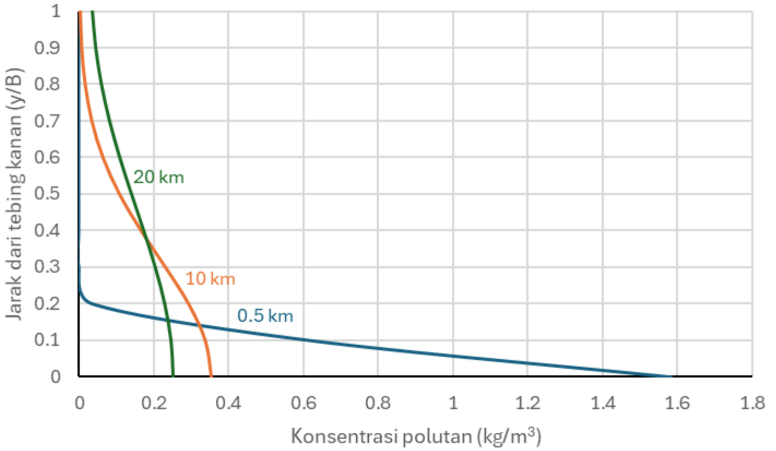


Konsentrasi polutan dalam satuan kg/m^3 yang berasal dari *diffuser* di tengah bentang sungai (program kontur ditulis dalam bahasa pemrograman Python)


diffuser di tengah bentang sungai
90.6 m
19 km
0.001
0.010
0.100
0.200
0.300
0.400
2.000
y/B
x/Ly


20 km
10 km
0.5 km
Jarak dari tebing kanan (y/B)
Konsentrasi polutan (kg/m³)


diffuser di tepi kanan sungai
94 km
90.6 m
0.001
0.010
0.100
0.200
0.300
0.400
0.900
0.800
2.000
y/B
x/Ly


20 km
10 km
0.5 km
Jarak dari tebing kanan (y/B)
Konsentrasi polutan (kg/m³)

Persebaran Polutan

Konveksi-Difusi Longitudinal

Konveksi-Difusi Longitudinal

- Persamaan konveksi-difusi dalam aliran turbulen

$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}+\frac{\partial vc}{\partial y}+\frac{\partial wc}{\partial z}=\frac{\partial}{\partial x}\left(\epsilon_{tx}\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right)$$

- Apabila
  - aliran hanya satu arah, $u \neq 0, v = w = 0$
  - difusi vertikal dan transversal telah dicapai, polutan telah menyebar di seluruh kedalaman dan lebar aliran → polutan telah menyebar di seluruh bagian tampang lintang aliran

Konveksi-Difusi Longitudinal

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \cancel{\frac{\partial vc}{\partial y}} + \cancel{\frac{\partial wc}{\partial z}} = \frac{\partial}{\partial x}\left(\epsilon_{tx}\frac{\partial c}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(\epsilon_{ty}\frac{\partial c}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(\epsilon_{tz}\frac{\partial c}{\partial z}\right)}$$

v = *w* = 0

difusi transversal telah dicapai

difusi vertikal telah dicapai

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} = \frac{\partial}{\partial x}\left(\epsilon_{tx}\frac{\partial c}{\partial x}\right) \Rightarrow \frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left\{\left(\overline{\epsilon_{tx}} + K'_x\right)\frac{\partial C}{\partial x}\right\}$$

$$\overline{\epsilon_{tx}} + K'_x = K_x$$

karena polutan telah menyebar di seluruh kedalaman aliran, maka tinjauan dilakukan untuk nilai rerata kedalaman

Konveksi-Difusi Longitudinal

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left\{\left(\overline{\epsilon_{tx}} + K_x'\right)\frac{\partial C}{\partial x}\right\} \Rightarrow \frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(\underbrace{K_x}_{\text{koefisien dispersi}}\frac{\partial C}{\partial x}\right)$$

- Di aliran permanen dan seragam, K_x konstan

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = K_x\frac{\partial^2 C}{\partial x^2} \Rightarrow \text{persamaan dispersi longitudinal}$$

- berlaku setelah
  - difusi vertikal di seluruh kedalaman aliran dicapai
  - difusi transversal di seluruh lebar aliran dicapai

Dispersi Longitudinal

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = K_x\frac{\partial^2 C}{\partial x^2}$$

Berlaku setelah jarak $L_y = \xi_y U \frac{B^2}{\overline{\overline{\epsilon_{ty}}}}$ atau setelah waktu $t_y = \xi_y \frac{B^2}{\overline{\overline{\epsilon_{ty}}}}$

di *far-field mixing zone*

- Koefisien dispersi, K_x

$K_x = 6(hu_*)$ → saluran tampang segiempat

$K_x = 0.011\frac{B^2U^2}{hu_*}$ → sungai

$140 < K_x < 500$ → saluran atau sungai yang memiliki distribusi kecepatan aliran ke arah vertikal dan ke arah transversal

Dispersi Longitudinal

- Jika polutan M_0 dimasukkan secara merata di tampang dan secara tiba-tiba, maka penyelesaian analitis persamaan dispersi longitudinal tsb adalah

$$C(x,t) = \frac{M_1}{\sqrt{4\pi K_x t}} \exp\left[-\frac{(x-Ut)^2}{4K_x t}\right]$$

$M_1 = M_0/S$ (kg/m²)

luas tampang aliran

$$C_{max}(t) = \frac{M_1}{\sqrt{4\pi K_x t}} = \frac{M_1}{\sqrt{4\pi K_x\, x/U}}$$

konsentrasi maksimum, bergerak dengan kecepatan U dan berkurang seiring waktu t

Dispersi Longitudinal

- Jika polutan sejumlah M_0 dimasukkan secara merata di tampang aliran selama waktu T
  - dapat dibaca sebagai satu seri polutan yang dimasukkan secara berurutan, masing-masing dalam waktu $\Delta\tau$ yang sangat kecil

$$\Delta C_i(x,t) = \frac{m_i}{S\sqrt{4\pi K_x (t-\tau_i)}} \exp\left\{-\frac{[x-U(t-\tau_i)]^2}{4K_x(t-\tau_i)}\right\} \qquad m_i = (M_0/T)\Delta\tau$$

$$C(x,t) = \sum_{i=1}^{n} \Delta C_i(x,t) = \frac{1}{S\sqrt{4\pi K_x}} \sum_{i=1}^{n} \frac{m_i}{\sqrt{(t-\tau_i)}} \exp\left\{-\frac{[x-U(t-\tau_i)]^2}{4K_x(t-\tau_i)}\right\}$$

Dispersi Longitudinal

- Jika polutan sejumlah M_0 dimasukkan secara merata di tampang aliran secara menerus (kontinu) dan konstan

$$C(x,t) = \frac{C_0}{2}\left[\exp\left(\frac{Ux}{K_x}\right)\operatorname{erfc}\left(\frac{x+Ut}{\sqrt{4K_x t}}\right) + \operatorname{erfc}\left(\frac{x-Ut}{\sqrt{4K_x t}}\right)\right]$$

C_0 konstanta

- Saat transpor telah mencapai permanen (*steady state condition*), $t \to \infty$
  - erfc(+∞) = 0
  - erfc(−∞) = 2

$$\frac{C}{C_0} = \begin{cases} 1 & U(x) > 0 \\ \exp\left(-\frac{Ux}{K_x}\right) & U(x) < 0 \end{cases}$$

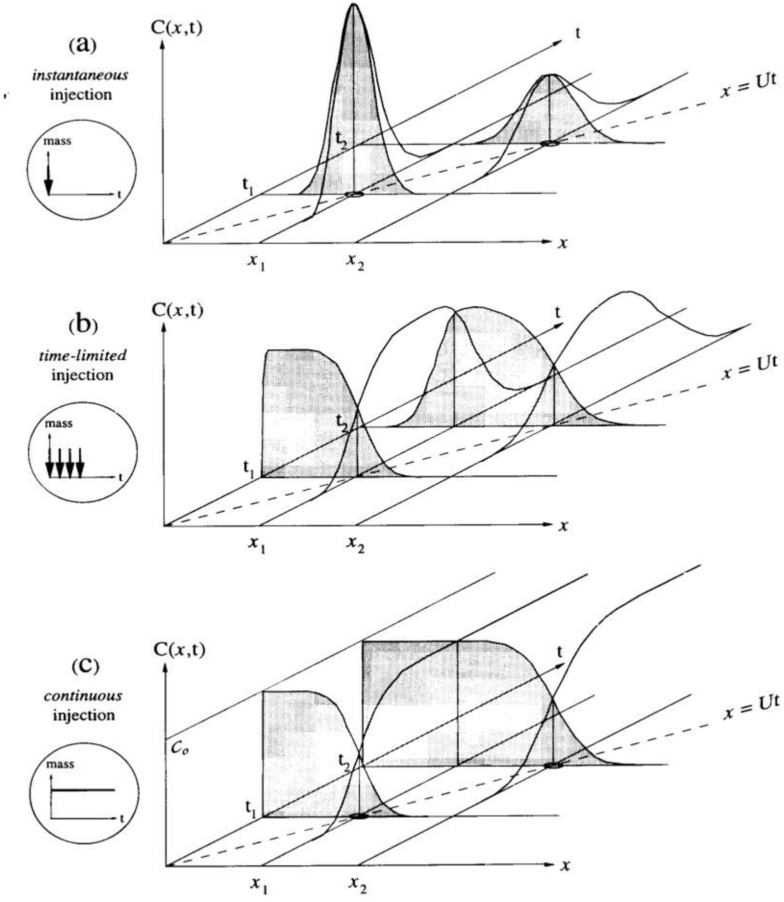
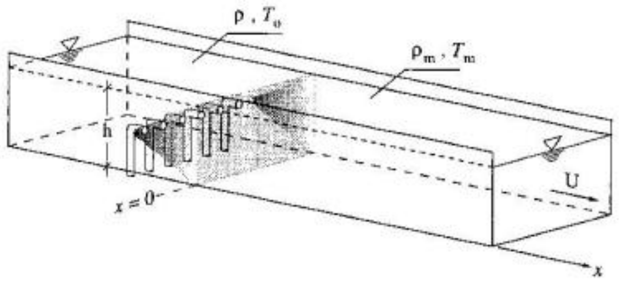


Fig. 8.11 Evolution of the concentration, $C(x,t)$, introduced uniformly over the cross section : a) *instantaneously*, b) *time-limited* and c) *continuously*.

Konveksi-difusi longitudinal

Ex. 8.D

In a large channel the water is flowing with a low average velocity of U = 0.1 [m/s] and the flow can be considered to be one-dimensional. The flow depth is h = 1.0 [m]. The density and the temperature of the water are ρ = 1000 [kg/m³] and T_0 = 15 [°C], respectively.



At a certain moment a brine solution is introduced continuously into the flow, and this at a constant rate by a multipoint-diffuser system spreading the mixture uniformly over the entire cross section of the channel almost instantaneously. The density and the temperature of the mixture at the cross section just at the downstream of the diffuser were measured as being ρ_m = 1010 [kg/m³] at T_m = 22 [°C]. The dispersion coefficient was obtained as being K_x = 0.1 [m²/s] by a previous experimental study.

Considering only the longitudinal dispersion in the *x*-direction, determine : *i)* the concentration distribution and *ii)* the temperature distribution at times t_1 = 1[h] and t_2 = 24 [h] after the start of the injection. *iii)* Subsequently calculate when the mixture with a density of ρ_m' = 1001 [kg/m³] arrives at a station located at 1 [km] downstream of the diffuser system.

Konveksi-difusi longitudinal

Ex. 8.E

The flow in a channel can be considered as turbulent and subcritical. The following hydraulic data have been measured : h = 0.7 [m], B = 6.0 [m], U = 1.0 [m/s] and $u_* = 0.07$ [m/s].

A certain quantity of salt, $M_0 = 4$ [kg] , is released into the flow and it spreads itself instantaneously over the entire cross section of the channel.

Study the temporal evolution of the concentration at a distance of $L_R = 1$ [km] downstream from the injection point for the following scenarios :

i) Scenario A : the salt is released instantaneously at the station $x = 0.0$ [m],

ii) Scenario B : the salt is released during a time period of T = 8 [min] at the station $x = 0.0$ [m],

iii) Scenario C : 2 [kg] of salt is released simultaneously at two stations located at $x_1 = 0.0$ [m] and $x_2 = 250.0$ [m].

iv) For the above three scenarios calculate the time period during which the salt concentration at $L_R = 1$ [km] is higher than $C' = 0.002$ [kg/m^3].

v) Calculate the concentration distribution at the time $t_R = 1000$ [s] for scenarios A and B.

UNIVERSITAS GADJAH MADA
DEPARTEMEN TEKNIK SIPIL DAN LINGKUNGAN
PRODI MAGISTER TEKNIK SIPIL

Dinamika Aliran dan Transfer Massa

Model Sebaran Polutan (*Softwares*)

Software Sebaran Polutan

- *Software* yang sebaiknya digunakan
  - Tidak berbayar
  - Dibuat oleh institusi yang terpercaya
  - Terus dikembangkan dan dirawat
  - Mudah dipakai

Software Sebaran Polutan di Sungai/Saluran

- Storm Water Management Model (SWMM) | US EPA
- HEC-RAS

  - *Software-software* itu menyelesaikan persamaan aliran dan transpor massa secara numerik, bukan secara analitik seperti yang dibahas dalam kuliah ini

Terima kasih

Dinamika Aliran dan Transfer Massa