# SISTEM PERSAMAAN LINEAR

*Systems of Linear Algebraic Equations*

# Sistem Persamaan Linear

❑ Acuan

   ❑ Chapra, S.C., Canale R.P., 1990, *Numerical Methods for Engineers*, 2nd Ed., McGraw-Hill Book Co., New York.

   ■ Chapter 7, 8, dan 9, hlm. 201-290.

# Sistem Persamaan Linear

❑ Serangkaian $n$ persamaan linear:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = c_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = c_2$$
$$\cdot$$
$$\cdot$$
$$\cdot$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = c_n$$

Sejumlah $n$ persamaan linear ini harus diselesaikan secara simultan untuk mendapatkan $x_1$, $x_2$,..., $x_n$ yang memenuhi setiap persamaan dalam sistem persamaan ini.

# Metode Penyelesaian

## Jml. pers. sedikit, $n$ «

- ❑ Penyelesaian
  - ❑ Grafis
  - ❑ Cramer
  - ❑ Eliminasi

## Jml. pers. banyak, $n$ »

- ❑ Penyelesaian langsung
  - ❑ Eliminasi Gauss
  - ❑ Gauss-Jordan
- ❑ Iteratif
  - ❑ Jacobi
  - ❑ Gauss-Seidel
  - ❑ *Successive Over Relaxation*

# Metode Grafis

# Metode Grafis

hampir sejajar

$x_2$

$a_{11}x_1 + a_{12}x_2 = c_1$

$a_{21}x_1 + a_{22}x_2 = c_2$

$x_1$

*ill-conditioned system*

berimpit

$x_2$

$a_{11}x_1 + a_{12}x_2 = c_1$

$a_{21}x_1 + a_{22}x_2 = c_2$

$x_1$

*singular system*

sejajar

$x_2$

$a_{11}x_1 + a_{12}x_2 = c_1$

$a_{21}x_1 + a_{22}x_2 = c_2$

$x_1$
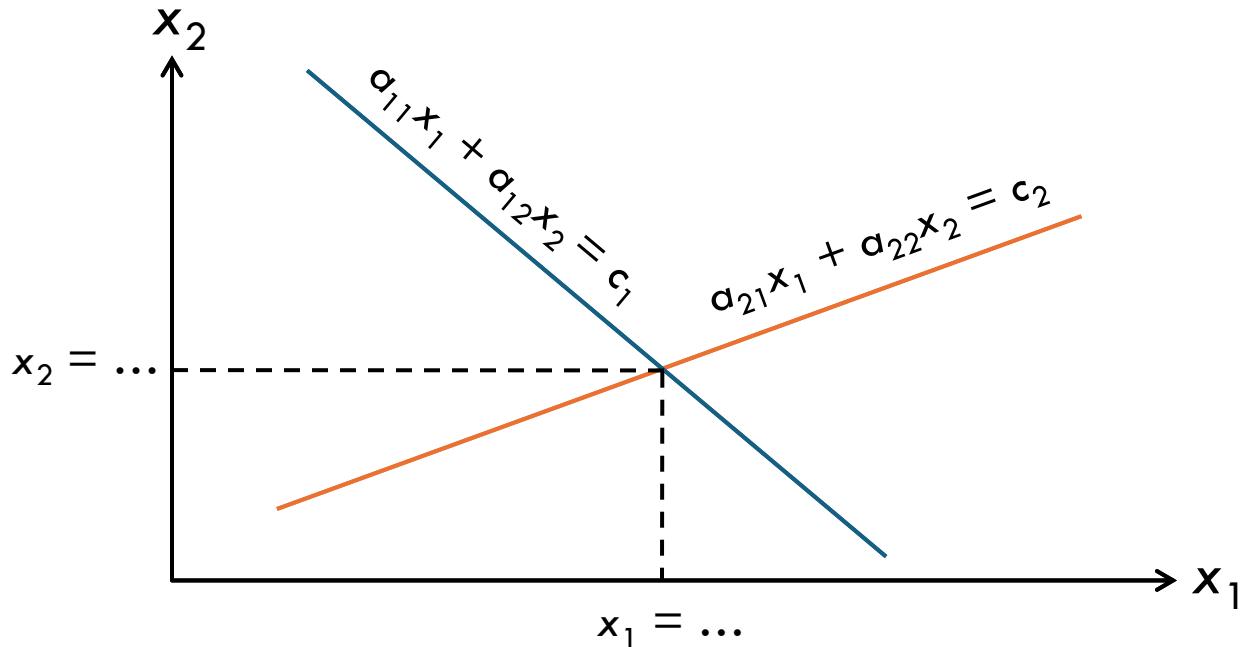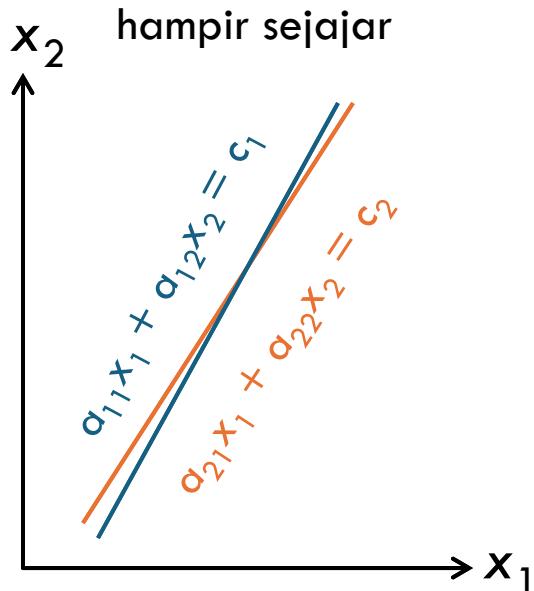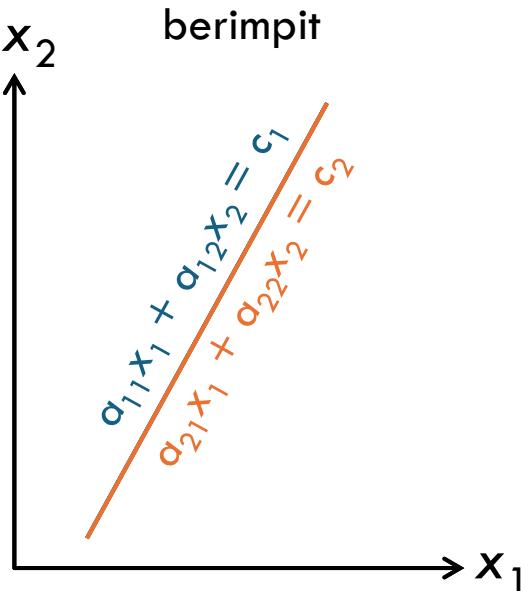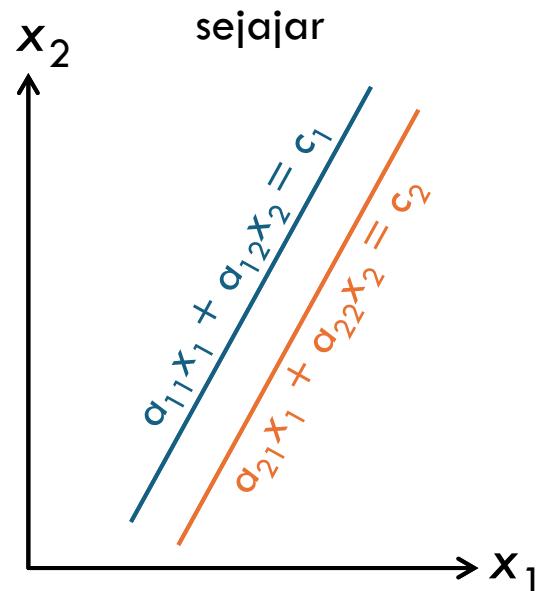
*singular system*

# Metode Cramer

❑ Variabel tak diketahui, $x_i$, merupakan perbandingan dua determinan matriks

  ❑ Penyebut   : determinan, $D$, matriks koefisien sistem persamaan

  ❑ Pembilang   : determinan matriks koefisien sistem persamaan seperti penyebut, namun koefisien kolom ke-$i$ diganti dengan koefisien $c_i$

❑ Contoh

  ❑ 3 persamaan linear
$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3$$

# Metode Cramer

$$[A] = \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad D = \det \boldsymbol{A} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$x_1 = \frac{\begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix}}{D} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix}}{D} \qquad x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}}{D}$$

# Determinan Matriks

❑ Matriks bujur sangkar: $n \times n$

❑ Mencari determinan matriks

   ❑ Hitungan manual

   ❑ MSExcel, dengan fungsi  =MDETERM()

❑ Contoh hitungan determinan matriks 2×2 dan 3×3

$$[A] = \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad [B] = \boldsymbol{B} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

# Determinan Matriks

$$D = \det \boldsymbol{A} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$D = \det \boldsymbol{B} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

# Metode Cramer

❑ Contoh: 3 persamaan linear

$$A \cdot X = C$$

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{Bmatrix}$$

$$\det A = 3\{7 \times 10 - (-0.3) \times (-0.2)\} + 0.1\{0.1 \times 10 - (-0.3) \times 0.3\} - 0.2\{0.1 \times (-0.2) - 7 \times 0.3\}$$

$$\det A = 210.353$$

# Metode Cramer

$$A_1 = \begin{bmatrix} 7.85 & -0.1 & -0.2 \\ -19.3 & 7 & -0.3 \\ 71.4 & -0.2 & 10 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 3 & 7.85 & -0.2 \\ 0.1 & -19.3 & -0.3 \\ 0.3 & 71.4 & 10 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 3 & -0.1 & 7.85 \\ 0.1 & 7 & -19.3 \\ 0.3 & -0.2 & 71.4 \end{bmatrix}$$

$$\det A_1 = |A_1| = 631.059 \qquad \det A_2 = |A_2| = -525.8825 \qquad \det A_3 = |A_3| = 1472.471$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{631.058}{210.353} = 3$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{-525.8825}{210.353} = -2.5$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{1472.471}{210.353} = 7$$

# Metode Eliminasi

❑ Contoh: 2 persamaan linear

$$a_{11}x_1 + a_{12}x_2 = c_1 \implies a_{21}(a_{11}x_1 + a_{12}x_2 = c_1) \implies a_{21}a_{11}x_1 + a_{21}a_{12}x_2 = a_{21}c_1$$

$$a_{21}x_1 + a_{22}x_2 = c_2 \implies a_{11}(a_{21}x_1 + a_{22}x_2 = c_2) \implies a_{11}a_{21}x_1 + a_{11}a_{22}x_2 = a_{11}c_2$$

$$a_{21}a_{12}x_2 - a_{11}a_{22}x_2 = a_{21}c_1 - a_{11}c_2$$

$$x_2 = \frac{a_{21}c_1 - a_{11}c_2}{a_{21}a_{12} - a_{11}a_{22}}$$

$$x_1 = \frac{c_1 - a_{12}x_2}{a_{11}} = \frac{c_1}{a_{11}} - \frac{a_{12}}{a_{11}}\left(\frac{a_{21}c_1 - a_{11}c_2}{a_{21}a_{12} - a_{11}a_{22}}\right)$$

# Eliminasi Gauss

❑ **Strategi**

  ❑ *Forward elimination*

  ❑ *Back substitution*

❑ **Contoh**

  ❑ 3 persamaan linear

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1 \quad (1)$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2 \quad (2)$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3 \quad (3)$$

# Eliminasi Gauss

❑ *Forward elimination #1*

   ❑ Hilangkan $x_1$ dari pers. kedua dan ketiga dengan operasi perkalian koefisien dan pengurangan terhadap pers. pertama.

*pivot equation*

*pivot coefficient*

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1 \quad (1)$$

$$\left(a_{22} - \frac{a_{21}}{a_{11}}a_{12}\right)x_2 + \left(a_{23} - \frac{a_{21}}{a_{11}}a_{13}\right)x_3 = c_2 - \frac{a_{21}}{a_{11}}c_1$$

$$a'_{22}x_2 + a'_{23}x_3 = c'_2 \quad (2')$$

$$\left(a_{32} - \frac{a_{31}}{a_{11}}a_{12}\right)x_2 + \left(a_{33} - \frac{a_{31}}{a_{11}}a_{13}\right)x_3 = c_3 - \frac{a_{31}}{a_{11}}c_1$$

$$a'_{32}x_2 + a'_{33}x_3 = c'_3 \quad (3')$$
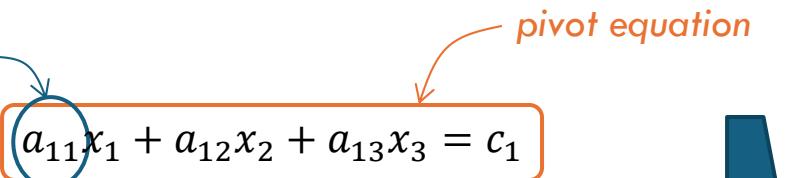
# Eliminasi Gauss

❑ *Forward elimination #2*

   ❑ Hilangkan $x_2$ dari pers. ketiga dengan operasi perkalian koefisien dan pengurangan terhadap pers. kedua.

*pivot coefficient*

*pivot equation*

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$$

$$a'_{22}x_2 + a'_{23}x_3 = c'_2$$

$$\left(a'_{33} - \frac{a'_{32}}{a'_{22}}a'_{23}\right)x_3 = c'_3 - \frac{a'_{32}}{a'_{22}}c'_2$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1 \quad (1)$$

$$a'_{22}x_2 + a'_{23}x_3 = c'_2 \quad (2')$$

$$a''_{33}x_3 = c''_3 \quad (3'')$$

# Eliminasi Gauss

❑ *Back substitution*

    ❑ Hitung $x_3$ dari pers. (3″), hitung $x_2$ dari pers. (2'), dan $x_1$ dari pers. (1)

$$x_3 = \frac{c_3''}{a_{33}''} \qquad \Longrightarrow \qquad x_2 = \frac{c_2' - a_{23}'x_3}{a_{22}'} \qquad \Longrightarrow \qquad x_1 = \frac{c_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$$

# Eliminasi Gauss

❑ *Forward elimination*

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = c_1$$
$$a'_{22}x_2 + a'_{23}x_3 + \cdots + a'_{2n}x_n = c'_2$$
$$a''_{23}x_3 + \cdots + a''_{3n}x_n = c''_3$$
$$\cdot$$
$$\cdot$$
$$\cdot$$
$$a_{nn}^{n-1}x_n = c_n^{n-1}$$

❑ *Back substitution*

$$x_n = \frac{c_n^{n-1}}{a_{nn}^{n-1}}$$

$$x_i = \frac{c_i^{i-1} - \sum_{j=i+1} a_{ij}^{i-1}x_j}{a_{ii}^{i-1}}, \quad i = n-1, n-2, \cdots, 1$$

# Eliminasi Gauss

❑ Contoh: 3 persamaan linear

$$(1) \quad 3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$(2) \quad 0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$(3) \quad 0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

# Eliminasi Gauss

**20**  https://istiarto.staff.ugm.ac.id

❑ *Forward elimination*

    ❑ Eliminasi $x_1$ dari Pers. 2 dan 3, Pers. 1 sebagai *pivot*

      (1)   $3x_1 - 0.1x_2 - 0.2x_3 = 7.85$

      (2′)  $0x_1 + 7.0033x_2 - 0.2933x_3 = -19.5617$

      (3′)  $0x_1 - 0.19x_2 + 10.02x_3 = 70.615$

    ❑ Eliminasi $x_2$ dari Pers. 3, Pers. 2 sebagai *pivot*

      (1)   $3x_1 - 0.1x_2 - 0.2x_3 = 7.85$

      (2′)  $0x_1 + 7.0033x_2 - 0.2933x_3 = -19.5617$

      (3″)  $0x_1 + 0x_2 + 10.0120x_3 = 70.0843$

# Eliminasi Gauss

❑ *Back substitution*

    ❑ Menghitung $x_3$ dari Pers. 3"

$$x_3 = \frac{70.0843}{10.0120} = 7$$

    ❑ Substitusi $x_3$ ke Pers. 2' untuk menghitung $x_2$

$$x_2 = \frac{-19.5617 + 0.2933 \times 7}{7.0033} = -2.5$$

    ❑ Substitusi $x_3$ dan $x_2$ ke Pers. 1 untuk menghitung $x_1$

$$x_1 = \frac{7.85 + 0.2 \times 7 + 0.1 \times (-2.5)}{3} = 3$$

# Metode Eliminasi

❑ Strategi

    ❑ Eliminasi variabel tak diketahui, $x_i$, dengan penggabungan dua persamaan.

    ❑ Hasil eliminasi adalah satu persamaan yang dapat diselesaikan untuk mendapatkan satu variabel $x_i$.

# Kelemahan Metode Eliminasi

❑ Pembagian dengan nol

    ❑ *Pivot coefficient* sama dengan nol ataupun sangat kecil.

    ❑ Pembagian dengan nol dapat terjadi selama proses eliminasi ataupun substitusi.

❑ *Round-off errors*

    ❑ Selama proses eliminasi maupun substitusi, setiap langkah hitungan bergantung pada langkah hitungan sebelumnya dan setiap kali terjadi kesalahan; kesalahan dapat berakumulasi, terutama apabila jumlah persamaan sangat banyak.

❑ *Ill-conditioned systems*

    ❑ *Ill-condition* adalah situasi dimana perubahan kecil pada satu atau beberapa koefisien berakibat perubahan yang besar pada hasil hitungan.

# Perbaikan

❑ Pemilihan *pivot* (*pivoting*)

    ❑ Urutan persamaan dipilih sedemikian hingga yang menjadi *pivot equation* adalah persamaan yang memberikan *pivot coefficient* terbesar.

# Metode Penyelesaian

❑ Matriks Inversi

  ❑ Gauss-Jordan

❑ Metode Iteratif

  ❑ Jacobi

  ❑ Gauss-Seidel

  ❑ *Successive Over-relaxation* (SOR)

# Metode Gauss-Jordan

❑ Mirip dengan metode eliminasi Gauss, tetapi tidak diperlukan *back substitution.*

❑ Contoh

   ❑ 3 persamaan linear

$$(1) \quad 3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$(2) \quad 0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$(3) \quad 0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

# Metode Gauss-Jordan

$$\begin{bmatrix} 3 & -0.1 & -0.2 & | & 7.85 \\ 0.1 & 7 & -0.3 & | & -19.3 \\ 0.3 & -0.1 & 10 & | & 71.4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & | & 2.6167 \\ 0.1 & 7 & -0.3 & | & -19.3 \\ 0.3 & -0.1 & 10 & | & 71.4 \end{bmatrix}$$
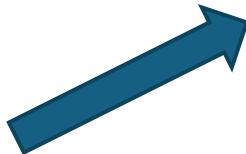
$$\begin{bmatrix} 3/3 & -0.1/3 & -0.2/3 & | & 7.85/3 \\ 0.1 & 7 & -0.3 & | & -19.3 \\ 0.3 & -0.1 & 10 & | & 71.4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & | & 2.6167 \\ 0 & 7.0033 & -0.2933 & | & -19.5617 \\ 0 & -0.1900 & 10.0200 & | & 70.6150 \end{bmatrix}$$

# Metode Gauss-Jordan

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & | & 2.6167 \\ 0 & 7.0033 & -0.2933 & | & -19.5617 \\ 0 & -0.1900 & 10.0200 & | & 70.6150 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & | & 2.6167 \\ 0/7.0033 & 7.0033/7.0033 & -0.2933/7.0033 & | & -19.5617/7.0033 \\ 0 & -0.1900 & 10.0200 & | & 70.6150 \end{bmatrix}$$
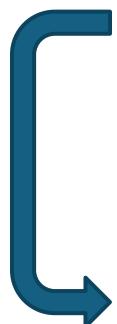
$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & | & 2.6167 \\ 0 & 1 & -0.0419 & | & -2.7931 \\ 0 & -0.1900 & 10.0200 & | & 70.6150 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -0.0681 & | & 2.5236 \\ 0 & 1 & -0.0419 & | & -2.7931 \\ 0 & 0 & 10.0120 & | & 70.0843 \end{bmatrix}$$

# Metode Gauss-Jordan

$$\begin{bmatrix} 1 & 0 & -0.0681 & 2.5236 \\ 0 & 1 & -0.0419 & -2.7931 \\ 0/10.0120 & 0/10.0120 & 10.0120/10.0120 & 70.0843/10.0120 \end{bmatrix}$$
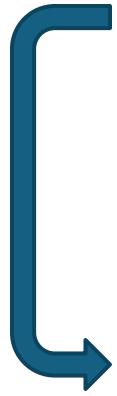
$$\begin{bmatrix} 1 & 0 & -0.0681 & 2.5236 \\ 0 & 1 & -0.0419 & -2.7931 \\ 0 & 0 & 1 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2.5 \\ 0 & 0 & 1 & 7 \end{bmatrix} \Rightarrow \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 3 \\ -2.5 \\ 7 \end{Bmatrix}$$

# Gauss-Jordan *vs* Eliminasi Gauss

❑ **Metode Gauss-Jordan**

   ❑ Jumlah operasi lebih banyak (50%)

   ❑ Memiliki kelemahan yang sama dengan eliminasi Gauss

      ■ Pembagian dengan nol

      ■ *Round-off error*

# Matriks Inversi

$$[A] \cdot \{X\} = \{C\} \Longrightarrow \{X\} = [A]^{-1} \cdot \{C\}$$

$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right] \implies \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & a_{11}^{-1} & a_{12}^{-1} & a_{13}^{-1} \\ 0 & 1 & 0 & a_{21}^{-1} & a_{22}^{-1} & a_{23}^{-1} \\ 0 & 0 & 1 & a_{31}^{-1} & a_{32}^{-1} & a_{33}^{-1} \end{array}\right]$$

# Matriks Inversi

❑ Contoh: 3 persamaan linear

$(1) \quad 3x_1 - 0.1x_2 - 0.2x_3 = 7.85$

$(2) \quad 0.1x_1 + 7x_2 - 0.3x_3 = -19.3$

$(3) \quad 0.3x_1 - 0.2x_2 + 10x_3 = 71.4$

# Matriks Inversi

$$[A] = \begin{bmatrix} 3 & -0.1 & -0.2 & | & 1 & 0 & 0 \\ 0.1 & 7 & -0.3 & | & 0 & 1 & 0 \\ 0.3 & -0.2 & 10 & | & 0 & 0 & 1 \end{bmatrix} \Longrightarrow [A] = \begin{bmatrix} 1 & -0.0333 & -0.0667 & | & 0.3333 & 0 & 0 \\ 0.1 & 7 & -0.3 & | & 0 & 1 & 0 \\ 0.3 & -0.2 & 10 & | & 0 & 0 & 1 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 1 & -0.0333 & -0.0667 & | & 0.3333 & 0 & 0 \\ 0 & 7.0033 & -0.2933 & | & -0.0333 & 1 & 0 \\ 0 & -0.1900 & 10.0200 & | & -0.0999 & 0 & 1 \end{bmatrix}$$

# Matriks Inversi

$$[A] = \begin{bmatrix} 1 & -0.0333 & -0.0667 & 0.3333 & 0 & 0 \\ 0 & 1 & -0.0417 & -0.0047 & 0.1422 & 0 \\ 0 & -0.1900 & 10.0200 & -0.0999 & 0 & 1 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 1 & 0 & -0.0681 & 0.3318 & 0.0047 & 0 \\ 0 & 1 & -0.0417 & -0.0047 & 0.1422 & 0 \\ 0 & 0 & 10.0121 & -0.1009 & 0.0270 & 1 \end{bmatrix}$$

# Matriks Inversi

$$[A] = \begin{bmatrix} 1 & 0 & -0.0681 \\ 0 & 1 & -0.0417 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{matrix} 0.3318 & 0.0047 & 0 \\ -0.0047 & 0.1422 & 0 \\ -0.0101 & 0.0027 & 0.0999 \end{matrix} \right]$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{matrix} 0.3325 & 0.0049 & 0.0068 \\ -0.0052 & 0.1423 & 0.0042 \\ -0.0101 & 0.0027 & 0.0999 \end{matrix} \right]$$

$$[A]^{-1}$$

# Matriks Inversi

$$\{X\} = [A]^{-1} \cdot \{C\}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 0.3325 & 0.0049 & 0.0068 \\ -0.0052 & 0.1423 & 0.0042 \\ -0.0101 & 0.0027 & 0.0999 \end{bmatrix} \cdot \begin{Bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 3.0004 \\ -2.4881 \\ 7.0002 \end{Bmatrix}$$

# Matriks Inversi

❑ Fungsi MS Excel untuk menghitung inversi sebuah matriks

   ❑ =MINVERSE()

❑ Fungsi MS Excel untuk menghitung perkalian dua buah matriks

   ❑ =MMULT()

❑ Ada kemungkinan matriks tidak memiliki inversi

# Metode Iteratif: Jacobi

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3$$

$$x_1^0 = 0$$
$$x_2^0 = 0$$
$$x_3^0 = 0$$

nilai awal,
biasanya $x_i^0 = 0$

iterasi diteruskan
sampai konvergen
$x_i^{n+1} \approx x_i^n, \forall x_i$
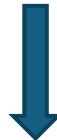
$$x_1 = \frac{c_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$$
$$x_2 = \frac{c_2 - a_{21}x_1 - a_{23}x_3}{a_{22}}$$
$$x_3 = \frac{c_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}$$

$$x_1^1 = \frac{c_1 - a_{12}x_2^0 - a_{13}x_3^0}{a_{11}}$$
$$x_2^1 = \frac{c_2 - a_{21}x_1^0 - a_{23}x_3^0}{a_{22}}$$
$$x_3^1 = \frac{c_3 - a_{31}x_1^0 - a_{32}x_2^0}{a_{33}}$$

$$x_1^{n+1} = \frac{c_1 - a_{12}x_2^n - a_{13}x_3^n}{a_{11}}$$
$$x_2^{n+1} = \frac{c_2 - a_{21}x_1^n - a_{23}x_3^n}{a_{22}}$$
$$x_3^{n+1} = \frac{c_3 - a_{31}x_1^n - a_{32}x_2^n}{a_{33}}$$

# Metode Iteratif: Jacobi

❑ Contoh: 3 persamaan linear

(1)    $3x_1 - 0.1x_2 - 0.2x_3 = 7.85$

(2)    $0.1x_1 + 7x_2 - 0.3x_3 = -19.3$

(3)    $0.3x_1 - 0.2x_2 + 10x_3 = 71.4$

# Metode Iteratif: Gauss-Seidel

$$x_1^1 = \frac{c_1 - a_{12}x_2^0 - a_{13}x_3^0}{a_{11}}$$

$$x_2^1 = \frac{c_2 - a_{21}x_1^0 - a_{23}x_3^0}{a_{22}}$$

$$x_3^1 = \frac{c_1 - a_{31}x_1^0 - a_{32}x_2^0}{a_{33}}$$

$$x_1^{n+1} = \frac{c_1 - a_{12}x_2^n - a_{13}x_3^n}{a_{11}}$$

$$x_2^{n+1} = \frac{c_2 - a_{21}x_1^{n+1} - a_{23}x_3^n}{a_{22}}$$

$$x_3^{n+1} = \frac{c_1 - a_{31}x_1^{n+1} - a_{32}x_2^{n+1}}{a_{33}}$$

iterasi diteruskan
sampai konvergen
$$x_i^{n+1} \approx x_i^n, \; \forall x_i$$

# Successive Over-relaxation Method

❑ Dalam setiap iterasi, nilai variabel terbaru (yang baru saja dihitung), $x_{n+1}$, tidak langsung dipakai dalam iterasi selanjutnya

❑ Dalam iterasi selanjutnya, nilai tsb dimodifikasi dengan memasukkan pengaruh nilai variabel lama (dalam iterasi sebelumnya), $x_n$

$$x_i^{new} = \lambda x_i^{n+1} + (1 - \lambda)x_i^n$$

   ❑ faktor relaksasi $\lambda$ dimaksudkan untuk mempercepat konvergensi hitungan (iterasi)

   ❑ *under-relaxation*:     $0 < \lambda < 1$

   ❑ *over-relaxation*:     $1 < \lambda < 2$

# Successive Over-relaxation Method

$$x_1^{n+1} = \frac{c_1 - a_{12}x_2^n - a_{13}x_3^n}{a_{11}}$$

$$x_2^{n+1} = \frac{c_2 - a_{21}[\lambda x_1^{n+1} + (1-\lambda)x_1^n] - a_{23}x_3^n}{a_{22}}$$

$$x_3^{n+1} = \frac{c_3 - a_{31}[\lambda x_1^{n+1} + (1-\lambda)x_1^n] - a_{32}[\lambda x_2^{n+1} + (1-\lambda)x_2^n]}{a_{33}}$$

# *Successive Over-relaxation Method*

❑ Contoh: 3 persamaan linear

(1)    $3x_1 - 0.1x_2 - 0.2x_3 = 7.85$
(2)   $0.1x_1 + 7x_2 - 0.3x_3 = -19.3$
(3)    $0.3x_1 - 0.2x_2 + 10x_3 = 71.4$

# Terima kasih

https://istiarto.staff.ugm.ac.id