UNIVERSITAS GADJAH MADA
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING
BACHELOR IN CIVIL ENGINEERING

Statistics and Probability

Hypothesis Testing

Hypothesis testing

- Mathematical model vs measurement
  - Comparison of theoretical line (computed by model) and measured values
    - If computed values match with measured ones, the model is accepted
    - If computed values do not fit to measured ones, the model is rejected
  - We have in many cases
    - Comparison of the computed and measured values cannot give clear clue whether to accept or to reject the model
    - Hypothesis testing provides an analysis tool in the comparison

Hypothesis testing

- Steps in making statistical tests
  - Formulate the hypothesis to be tested
  - Formulate an alternative hypothesis
  - Define a test statistic
  - Define the distribution of the test statistic
  - Define the rejection region or critical region of the test statistic
  - Collect the data needed to calculate the test statistic
  - Determine if the calculated value of the test statistic falls in the rejection region of the distribution of the test statistic

Errors in hypothesis testing

decision	hypothesis is true	hypothesis is false
accept hypothesis	correct decision	Type II error, β
reject hypothesis	Type I error, α	correct decision

α is the probability of committing error type I
β is the probability of committing error type II

α and β have to be small
α is more important than β

Notation

- H_0 = null hypothesis (hypothesis being tested)
- H_1 = alternative hypothesis
- $(1 - \alpha)$ = confidence level
- α = level of significance

Hypothesis testing on mean

$H_0: \mu = \mu_1$
$H_1: \mu = \mu_2$

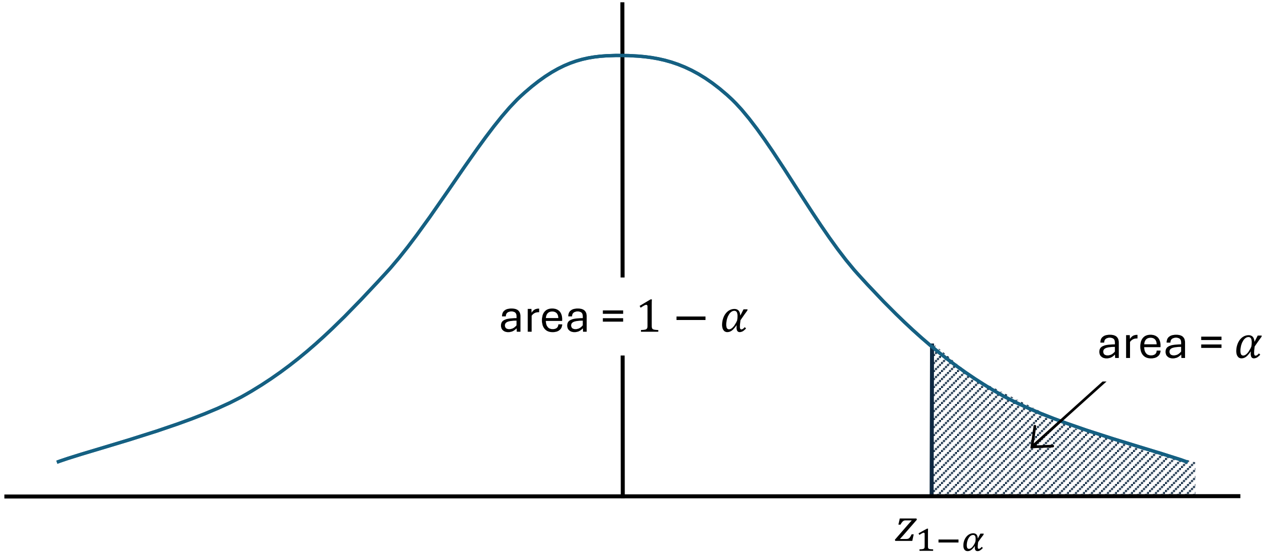
normal distribution
σ_X^2 is known → it is a rare case

Test statistic $Z = \frac{\sqrt{n}}{\sigma_X}(\bar{X} - \mu_1)$ has a standard normal distribution

If $\mu_1 > \mu_2 \Longrightarrow H_0$ is rejected if $\bar{X} \leq \mu_1 - z_{1-\alpha}\frac{\sigma_X}{\sqrt{n}} \Longrightarrow Z \leq -z_{1-\alpha}$

If $\mu_1 < \mu_2 \Longrightarrow H_0$ is rejected if $\bar{X} \leq \mu_1 + z_{1-\alpha}\frac{\sigma_X}{\sqrt{n}} \Longrightarrow Z \geq z_{1-\alpha}$

Hypothesis testing on mean



$$\text{prob}(Z > z_{1-\alpha}) = \alpha$$

Hypothesis testing on mean

$H_0: \mu = \mu_1$
$H_1: \mu = \mu_2$

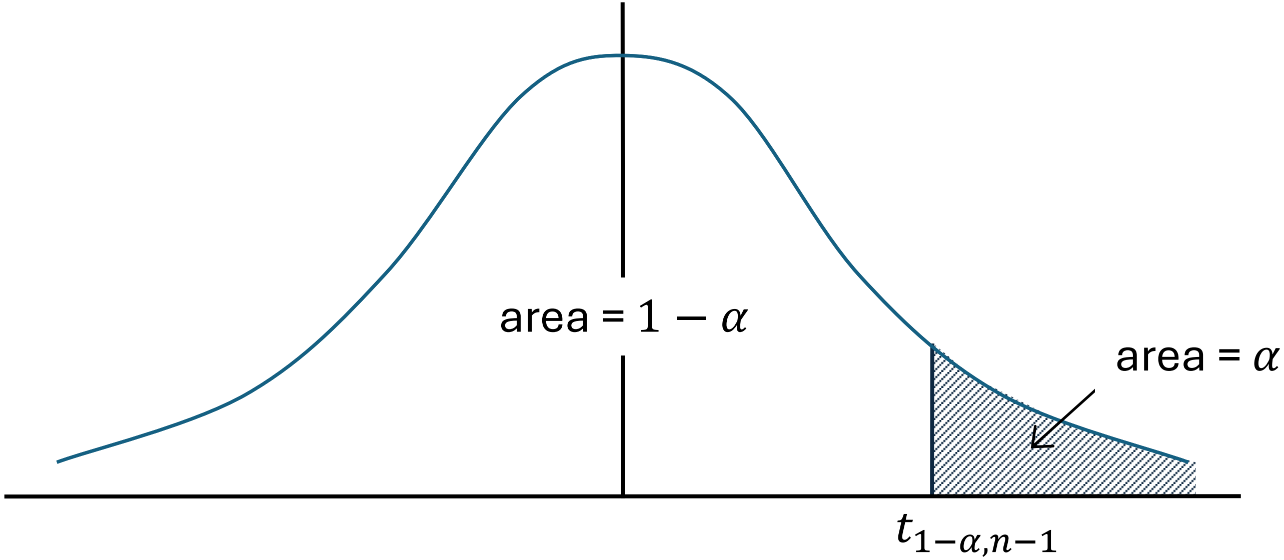
normal distribution
σ_X^2 is unknown

Test statistic $T = \frac{\sqrt{n}}{s_X}(\bar{X} - \mu_1)$ has a *t* distribution with $(n-1)$ degrees of freedom

If $\mu_1 > \mu_2 \Rightarrow H_0$ is rejected if $\bar{X} \le \mu_1 - t_{1-\alpha,n-1}\frac{s_X}{\sqrt{n}} \Rightarrow T \le -t_{1-\alpha,n-1}$

If $\mu_1 < \mu_2 \Rightarrow H_0$ is rejected if $\bar{X} \ge \mu_1 + t_{1-\alpha,n-1}\frac{s_X}{\sqrt{n}} \Rightarrow T \ge -t_{1-\alpha,n-1}$

Hypothesis testing on mean



$$\text{prob}\left(T > t_{1-\alpha,n-1}\right) = \alpha$$

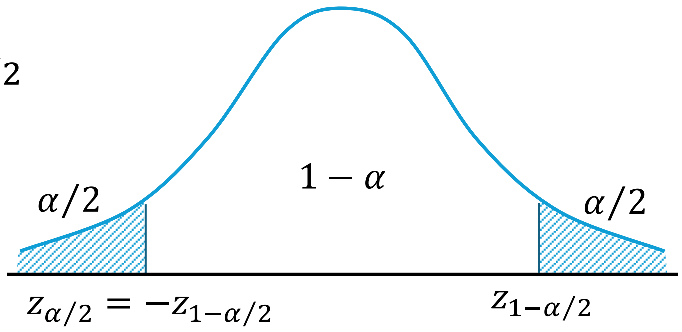
Hypothesis testing on mean

$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$

normal distribution
σ_X^2 is known → it is a rare case

Test statistic $Z = \frac{\sqrt{n}}{\sigma_X}(\bar{X} - \mu_0)$ has a standard normal distribution

H_0 is rejected if $|Z| = \left|\frac{\sqrt{n}}{\sigma_X}(\bar{X} - \mu_0)\right| > z_{1-\alpha/2}$



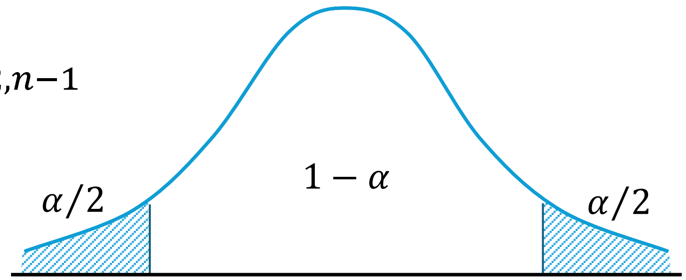
Hypothesis testing on mean

$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$

normal distribution
σ_X^2 is unknown

Test statistic $T = \frac{\sqrt{n}}{s_X}(\bar{X} - \mu_0)$ has a *t* distribution with $(n-1)$ degrees of freedom

H_0 is rejected if $|T| = \left|\frac{\sqrt{n}}{s_X}(\bar{X} - \mu_0)\right| > t_{1-\alpha/2,n-1}$



Hypothesis testing on mean

- Result of a hypothesis testing
  - accept H_0
  - fail to reject H_0
- Meaning
  - H_0: $\mu = \mu_0$
  - Accepting H_0 means that
    - we fail to reject H_0 → we say that based on the sample that we have, we say that the population mean is not significantly different from μ_0
    - we cannot say that the population mean really equals to μ_0 since we do not prove that $\mu = \mu_0$

Test for differences in means of two normal distributions

$H_0\colon\ \mu_1 - \mu_2 = \delta$

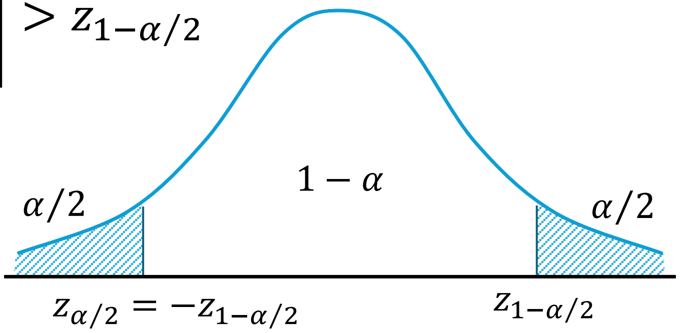
$H_1\colon \mu_1 - \mu_2 \neq \delta$

normal distribution

var(X) and var(X) are known → it is a rare case

Test statistic $$Z = \frac{\left(\bar{X}_1 - \bar{X}_2 - \delta\right)}{\left({\sigma_1}^2/n_1 + {\sigma_2}^2/n_2\right)^{1/2}}$$ has a standard normal distribution

H_0 is rejected if $$|Z| = \left|\frac{\left(\bar{X}_1 - \bar{X}_2 - \delta\right)}{\left({\sigma_1}^2/n_1 + {\sigma_2}^2/n_2\right)^{1/2}}\right| > z_{1-\alpha/2}$$



Test for differences in means of two normal distributions

$H_0\colon \mu_1 - \mu_2 = \delta$

$H_1\colon \mu_1 - \mu_2 \neq \delta$

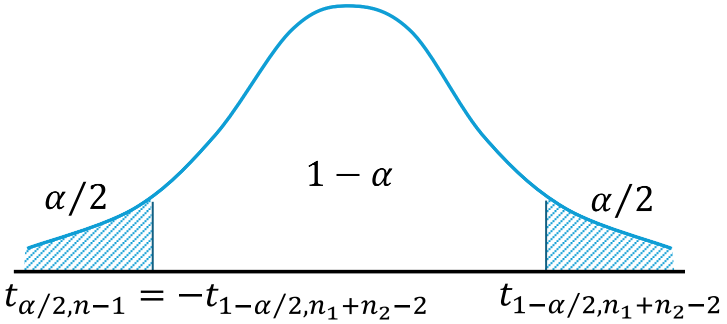
normal distribution

$\sigma_X{}^2$ is unknown

Test statistic $$T = \frac{\left(\bar{X}_1 - \bar{X}_2 - \delta\right)}{\left\{\frac{(n_1 + n_2)\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right]}{[n_1 n_2 (n_1 + n_2 - 2)]}\right\}^{1/2}}$$

has a t distribution with $(n_1 + n_2 - 2)$ degrees of freedom

H_0 is rejected if $|T| > t_{1-\alpha/2, n_1+n_2-2}$

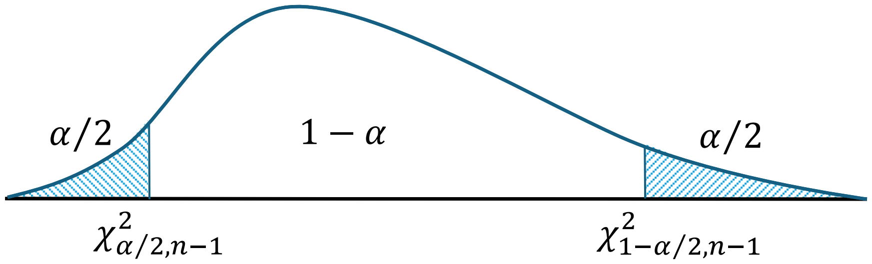


Test of variance

$H_0: \sigma^2 = \sigma_0^2$
$H_1: \sigma^2 \neq \sigma_0^2$
} normal distribution

Test statistic $\chi_c^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma_0^2}$ has a chi-square distribution

H_0 is accepted if $\chi^2_{\alpha/2,n-1} < \chi_c^2 < \chi^2_{1-\alpha/2,n-1}$

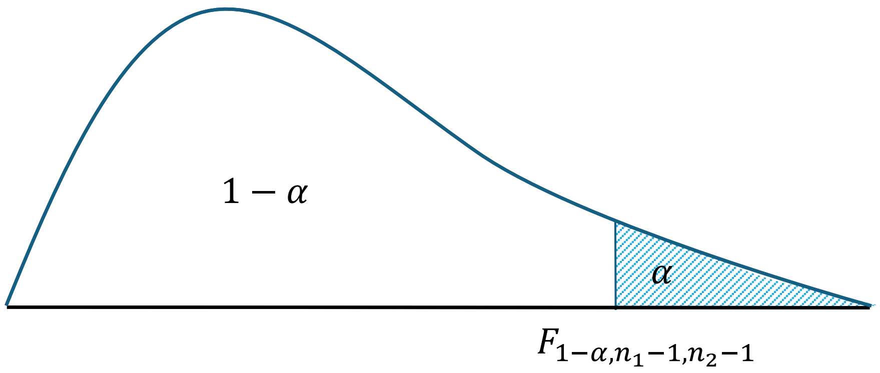


Test on variance of two normal distributions

$H_0: {\sigma_1}^2 = {\sigma_2}^2$
$H_1: {\sigma_1}^2 \neq {\sigma_2}^2$
} normal distribution

Test statistic $F_c = \frac{{s_1}^2}{{s_2}^2}$ has an F distribution with $(n_1 - 1)$ and $(n_2 - 1)$ degrees of freedom and ${s_1}^2 > {s_2}^2$

H_0 is rejected if $F_c > F_{1-\alpha, n_1-1, n_2-1}$



Test on variance of several normal distributions

H_0: ${\sigma_1}^2 = {\sigma_2}^2 = \cdots = {\sigma_k}^2$

H_1: ${\sigma_1}^2 \neq {\sigma_2}^2 \neq \cdots \neq {\sigma_k}^2$

} normal distribution

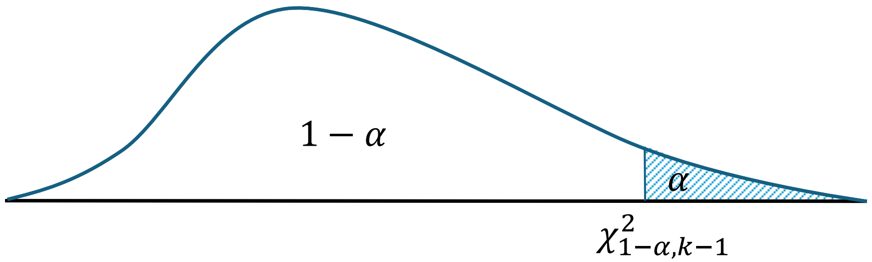
Test statistic Q/h has an chi-square distribution with $(k-1)$ degrees of freedom

$$Q = \sum_{i=1}^{k} (n-1) \ln \left[\sum_{i=1}^{k} \frac{(n_1 - 1){s_i}^2}{N-k} \right] - \sum_{i=1}^{k} (n-1) \ln {s_i}^2$$

$$h = 1 + \frac{1}{3(k-1)} \left[\sum_{i=1}^{k} \left(\frac{1}{n_i - 1} \right) - \frac{1}{N-k} \right]$$

$$N = \sum_{i=1}^{k} n_i$$

H_0 is rejected if $Q/h > \chi^2_{1-\alpha,k-1}$



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