BIVARIATE PROBABILITY DISTRIBUTION

CONTOH

Tabel di bawah ini menunjukkan jumlah event (hari) yang memiliki temperatur rerata harian (T) dan kelembaban relatif rerata (H) yang dipetik dari data harian periode 8 hari selama 43 tahun.

		Temperature, T (°F)					
		20–30	30–40	40–50	50–60	60–70	70–80
Relative Humidity, H (%)	0–20	2	4	6	2	2	1
	20–40	4	8	12	30	6	9
	40–60	5	15	30	60	30	20
	60–80	3	7	9	25	17	11
	80–100	1	0	2	12	8	3

Carilah:

1. $f_{T,H}(t_i, h_i)$
2. $f_T(t_i)$ dan $F_T(t_i)$
3. $f_H(h_j)$ dan $F_H(h_j)$
4. Probability:
   a. $40 \le T \le 50$ dan $60 \le H \le 80$
   b. $40 \le T \le 50$ pada saat $60 \le H \le 80$
   c. $T \le 60$
   d. $H \le 60$
   e. $T \le 40$ dan $H \le 40$
5. Apakah T dan H independent?

PENYELESAIAN

Temperatur: T → 6 interval: t_i, $i = 1, 2, \ldots, 6$

Kelembaban: H → 5 interval: h_j, $j = 1, 2, \ldots, 5$

$n_{i,j}$ = jumlah 8-hari sesuai dengan interval i, j

$$N = \sum_{i,j} n_{i,j} = 344$$

$$f_{T,H}(t_i, h_j) = \frac{n_{i,j}}{N}$$

Hitungan frekuensi setiap nilai temperatur dan kelembaban relatif disajikan pada tabel di bawah ini.

j	i						$f_H(h_j)$	$F_H(h_j)$
	20–30 (1)	30–40 (2)	40–50 (3)	50–60 (4)	60–70 (5)	70–80 (6)		
0–20 (1)	0.0058	0.0116	0.0174	0.0058	0.0058	0.0029	0.0494	0.0494
20–40 (2)	0.0116	0.0233	0.0349	0.0872	0.0174	0.0262	0.2006	0.2500
40–60 (3)	0.0145	0.0436	0.0872	0.1744	0.0872	0.0581	0.4651	0.7151
60–80 (4)	0.0087	0.0203	0.0262	0.0727	0.0494	0.0320	0.2093	0.9244
80–100 (5)	0.0029	0.0000	0.0058	0.0349	0.0233	0.0087	0.0756	**1.0000**
$f_T(t_i)$	0.0436	0.0988	0.1715	0.3750	0.1831	0.1279		
$F_T(t_i)$	0.0436	0.1424	0.3140	0.6890	0.8721	**1.0000**		

Joint and conditional probabilities (untuk discrete random variables)

- $\text{prob}(40 \le T \le 50 \text{ dan } 60 \le H \le 80) = f_{T,H}(t_3, h_4) = 0.0262$
- $\text{prob}(40 \le T \le 50 \text{ pada saat } 60 \le H \le 80) = f_{T,H}(t_3 | h_4)$

$$f_{T,H}(t_3|h_4) = \frac{f_{T,H}(t_3,h_4)}{f_H(h_4)} = \frac{0.0262}{0.2093} = 0.125$$

- $\text{prob}(T \le 60) = F_T(60) = F_T(t_4) = 0.6889$
- $\text{prob}(H \le 60) = F_H(60) = F_H(h_3) = 0.7151$
- $\text{prob}(T \le 40 \text{ dan } H \le 40) = F_{T,H}(t_2, h_2)$

$$F_{T,H}(t_2,h_2) = \sum_{i=1}^{2}\sum_{j=1}^{2} f_{T,H}(t_i,h_j) = 0.0522$$

- Jika *T* dan *H* independent, maka $f_T(t_i) \cdot f_H(h_j) = f_{T,H}(t_i, h_j)$
  $i = 3$ dan $j = 3$:

$$\left.\begin{array}{l} f_T(t_3) = 0.1715 \\ f_H(h_3) = 0.4651 \end{array}\right\rangle \; f_{T,H}(t_3,h_3) = 0.0872 \quad \Rightarrow \quad f_T(t_3)\cdot f_H(h_3) \ne f_{T,H}(t_3,h_3)$$

  dengan demikian *T* dan *H* dependent.