



UNIVERSITAS GADJAH MADA
DEPARTEMEN TEKNIK SIPIL DAN LINGKUNGAN
PRODI MAGISTER TEKNIK SIPIL

Statistika Teknik

Uji Hipotesis

Uji Hipotesis

- Model Matematika vs Pengukuran
 - komparasi garis teoretik (prediksi menurut model) dan data pengukuran
 - jika prediksi model sesuai dengan data pengukuran, maka model diterima
 - jika prediksi model menyimpang dari data pengukuran, maka model ditolak
- Dalam sejumlah kasus, yang terjadi adalah
 - hasil komparasi prediksi model dan data pengukuran tidak cukup jelas untuk menyatakan bahwa model diterima atau ditolak
 - uji hipotesis sebagai alat analisis dalam komparasi tersebut

Prosedur Uji Hipotesis

- Rumuskan hipotesis
- Rumuskan hipotesis alternatif
- Tetapkan statistika uji
- Tetapkan distribusi statistika uji
- Tentukan nilai kritik sebagai batas statistika uji harus ditolak
- Kumpulkan data untuk menyusun statistika uji
- Kontrol posisi statistika uji terhadap nilai kritis

Kemungkinan Kesalahan

Pilihan	Keadaan Nyata	
	Hipotesis Benar	Hipotesis Salah
Menerima	Tidak Salah	Kesalahan Tipe II
Menolak	Kesalahan Tipe I	Tidak Salah

Notasi

H_0 adalah hipotesis (yang diuji)

H_a adalah hipotesis alternatif

$1-\alpha$ adalah tingkat keyakinan (*confidence level*)

Uji Hipotesis Nilai Rata-rata

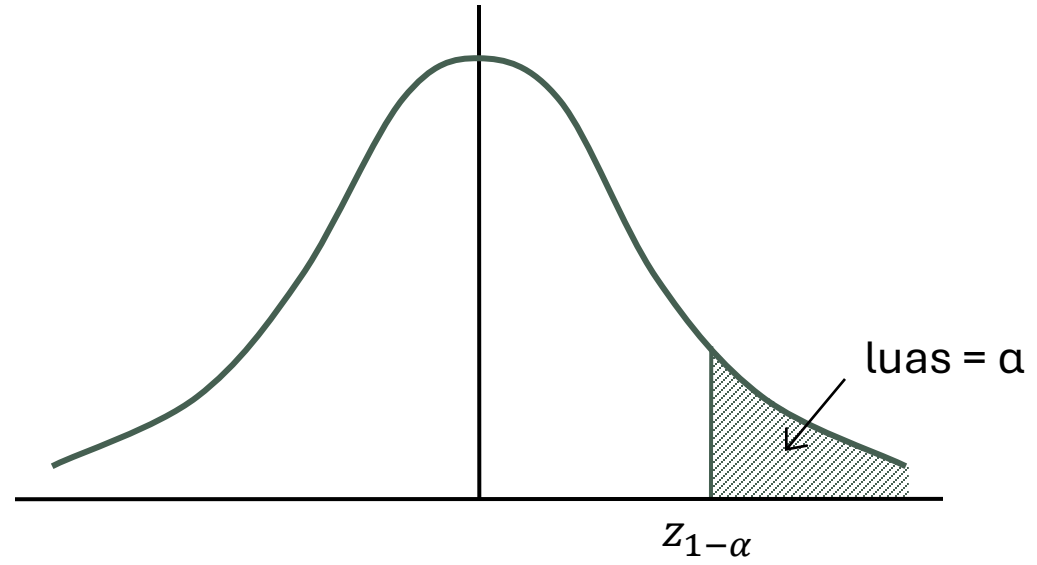
$$\left. \begin{array}{l} H_0 : \mu = \mu_1 \\ H_a : \mu = \mu_2 \end{array} \right\} \begin{array}{l} \text{Distribusi Normal} \\ \sigma_X^2 \text{ diketahui} \end{array}$$

tingkat keyakinan $1 - \alpha$

Statistik uji $Z = \frac{\sqrt{n}}{\sigma_X} (\bar{X} - \mu_1)$ berdistribusi normal

Jika $\mu_1 > \mu_2$, H_0 ditolak jika $\bar{X} < \mu_1 - z_{1-\alpha} \frac{\sigma_X}{\sqrt{n}} \Rightarrow Z < -z_{1-\alpha}$

Jika $\mu_1 < \mu_2$, H_0 ditolak jika $\bar{X} > \mu_1 + z_{1-\alpha} \frac{\sigma_X}{\sqrt{n}} \Rightarrow Z > z_{1-\alpha}$



$$\text{prob}(Z > z_{1-\alpha}) = \alpha$$

Uji Hipotesis Nilai Rata-rata

$$\left. \begin{array}{l} H_0 : \mu = \mu_1 \\ H_a : \mu = \mu_2 \end{array} \right\} \begin{array}{l} \text{Distribusi Normal} \\ \sigma_X^2 \text{ tidak diketahui} \end{array}$$

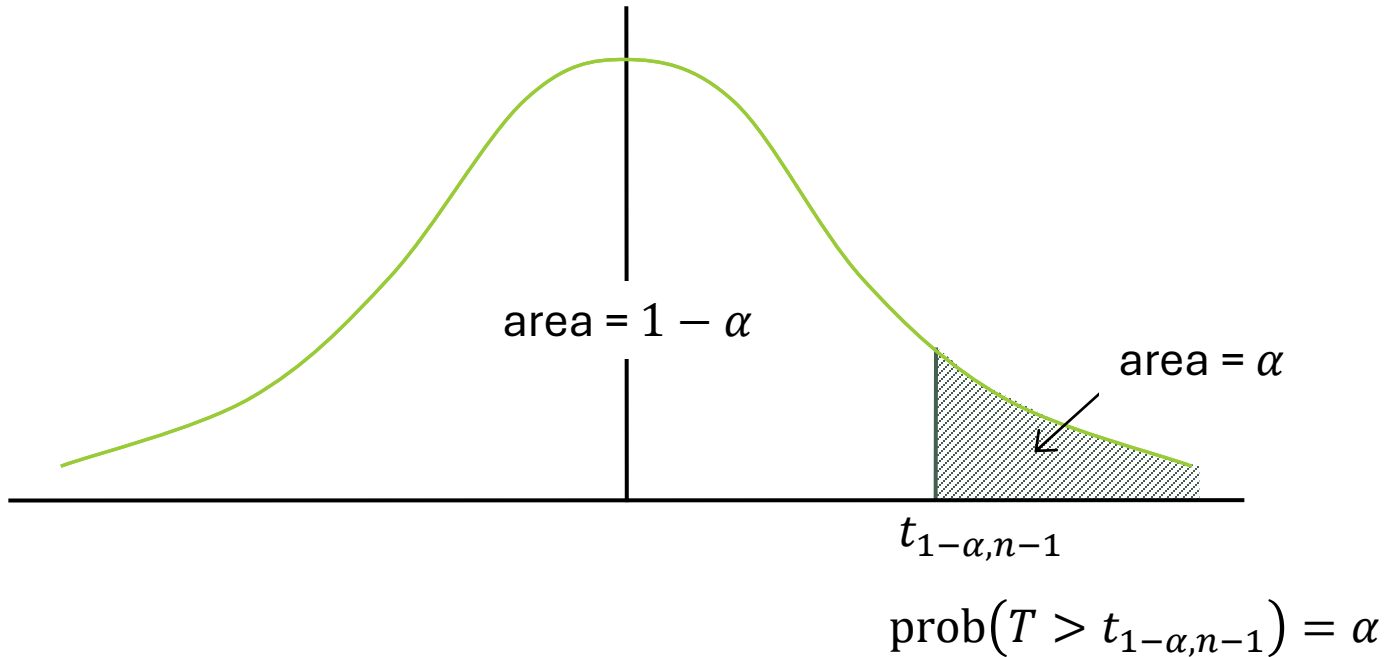
tingkat keyakinan $1 - \alpha$

Statistik uji $T = \frac{\sqrt{n}}{s_X} (\bar{X} - \mu_1)$ berdistribusi t

Jika $\mu_1 > \mu_2$, H_0 ditolak jika $\bar{X} < \mu_1 - t_{1-\alpha, n-1} \frac{s_X}{\sqrt{n}} \Rightarrow T < -t_{1-\alpha, n-1}$

Jika $\mu_1 < \mu_2$, H_0 ditolak jika $\bar{X} > \mu_1 + t_{1-\alpha, n-1} \frac{s_X}{\sqrt{n}} \Rightarrow T > t_{1-\alpha, n-1}$

Uji Hipotesis Nilai Rerata



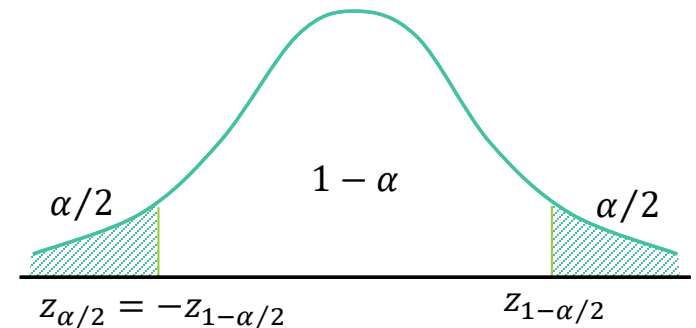
Uji Hipotesis Nilai Rata-rata

$$\left. \begin{array}{l} H_0 : \mu = \mu_0 \\ H_a : \mu \neq \mu_0 \end{array} \right\} \begin{array}{l} \text{Distribusi Normal} \\ \sigma_X^2 \text{ diketahui} \end{array}$$

tingkat keyakinan $1 - \alpha$

Statistik uji $Z = \frac{\sqrt{n}}{\sigma_X} (\bar{X} - \mu_0)$ berdistribusi normal

H_0 ditolak jika $|Z| = \left| \frac{\sqrt{n}}{\sigma_X} (\bar{X} - \mu_0) \right| > z_{1-\alpha/2}$



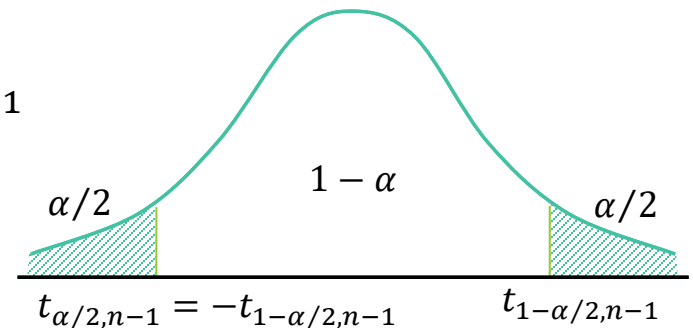
Uji Hipotesis Nilai Rata-rata

$$\left. \begin{array}{l} H_0 : \mu = \mu_0 \\ H_a : \mu \neq \mu_0 \end{array} \right\} \begin{array}{l} \text{Distribusi Normal} \\ \sigma_X^2 \text{ tidak diketahui} \end{array}$$

tingkat keyakinan $1 - \alpha$

Statistik uji $T = \frac{\sqrt{n}}{s_X} (\bar{X} - \mu_0)$ berdistribusi t

H_0 ditolak jika $|T| = \left| \frac{\sqrt{n}}{s_X} (\bar{X} - \mu_0) \right| > t_{1-\alpha/2, n-1}$



Uji Hipotesis Nilai Rata-rata

- Hasil uji hipotesis adalah
 - menolak H_0 , atau
 - tidak menolak H_0
- Artinya
 - $H_0: \mu = \mu_0$
 - Tidak menolak $H_0 \rightarrow$ “menerima” H_0 berarti bahwa μ tidak berbeda secara signifikan dengan μ_0 .
 - Tetapi tidak dikatakan bahwa μ benar-benar sama dengan μ_0 karena kita tidak membuktikan bahwa $\mu = \mu_0$.

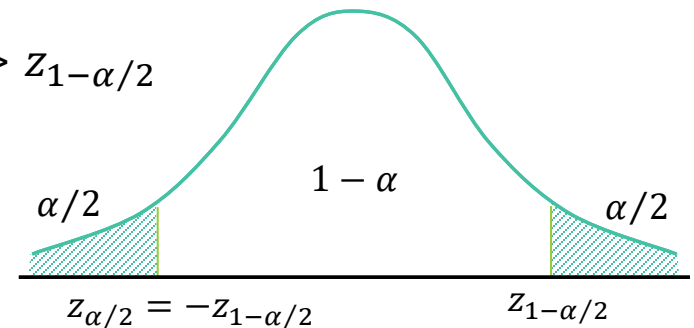
Uji hipotesis beda nilai rata-rata dua buah distribusi normal

$$\left. \begin{array}{l} H_0 : \mu_1 - \mu_2 = \delta \\ H_a : \mu_1 - \mu_2 \neq \delta \end{array} \right\} \begin{array}{l} \text{Distribusi Normal} \\ \text{var}(X_1) \text{ dan } \text{var}(X_2) \text{ diketahui} \end{array}$$

tingkat keyakinan $1 - \alpha$

Statistik uji $Z = \frac{\bar{X}_1 - \bar{X}_2 - \delta}{(\sigma_1^2/n_1 + \sigma_2^2/n_2)^{1/2}}$ berdistribusi normal

H_0 ditolak jika $|Z| = \left| \frac{\bar{X}_1 - \bar{X}_2 - \delta}{(\sigma_1^2/n_1 + \sigma_2^2/n_2)^{1/2}} \right| > z_{1-\alpha/2}$



Uji hipotesis beda nilai rata-rata dua buah distribusi normal

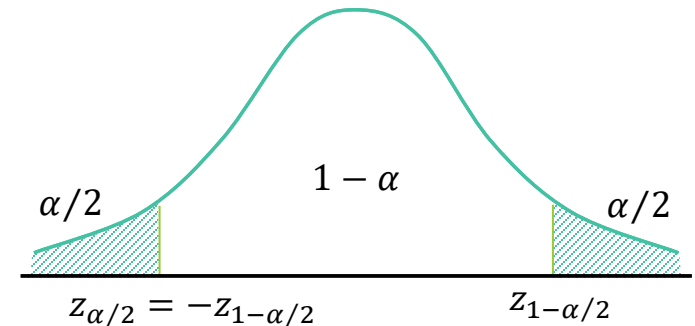
$$\left. \begin{array}{l} H_0 : \mu_1 - \mu_2 = \delta \\ H_a : \mu_1 - \mu_2 \neq \delta \end{array} \right\} \begin{array}{l} \text{Distribusi Normal} \\ \text{var}(X_1) \text{ dan } \text{var}(X_2) \text{ tidak diketahui} \end{array}$$

tingkat keyakinan $1 - \alpha$

$$\text{Statistik uji } T = \frac{\bar{X}_1 - \bar{X}_2 - \delta}{\left\{ \frac{(n_1 + n_2)[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2]}{[n_1 n_2 (n_1 + n_2 - 2)]} \right\}^{1/2}}$$

berdistribusi t
dengan $n_1 + n_2 - 2$
degrees of freedom

H_0 ditolak jika $|T| > t_{1-\alpha/2, n_1+n_2-2}$



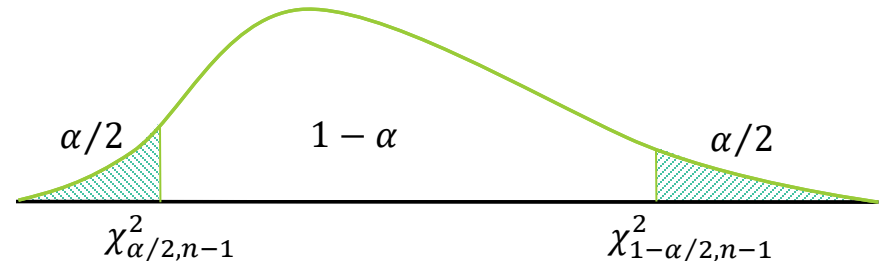
Uji Hipotesis Nilai Varians

$$\left. \begin{array}{l} H_0 : \sigma^2 = \sigma_0^2 \\ H_a : \sigma^2 \neq \sigma_0^2 \end{array} \right\} \text{Distribusi Normal}$$

tingkat keyakinan $1 - \alpha$

Statistik uji $\chi_c^2 = \sum_{i=1}^n \frac{(x_i - \bar{X})^2}{\sigma_0^2}$ berdistribusi chi-kuadrat

H_0 diterima (**tidak** ditolak) jika $\chi_{\alpha/2, n-1}^2 < \chi_c^2 < \chi_{1-\alpha/2, n-1}^2$



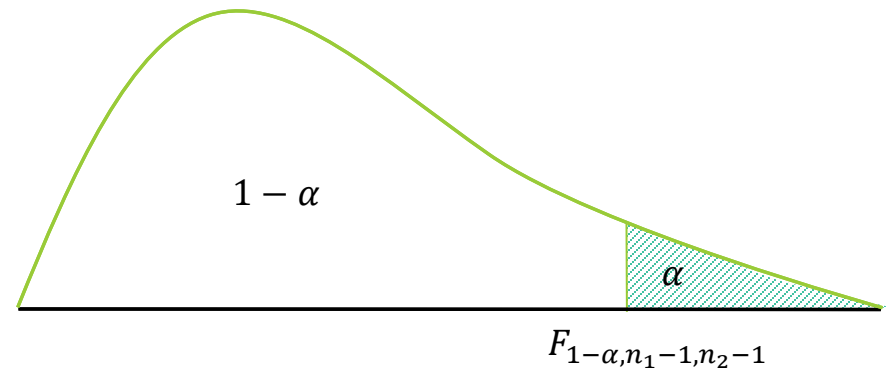
Uji Hipotesis Nilai Varians

$$\left. \begin{array}{l} H_0 : \sigma_1^2 = \sigma_2^2 \\ H_a : \sigma_1^2 \neq \sigma_2^2 \end{array} \right\} \text{ 2 Distribusi Normal}$$

tingkat keyakinan $1 - \alpha$

Statistik uji $F_c = \frac{s_1^2}{s_2^2}$ berdistribusi F dengan
 $(n_1 - 1)$ dan $(n_2 - 1)$ *degrees of freedom*
 $s_1^2 > s_2^2$

H_0 ditolak jika $F_c > F_{1-\alpha, n_1-1, n_2-1}$



Uji Hipotesis Nilai Varians

$$\left. \begin{array}{l} H_0 : \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2 \\ H_a : \sigma_1^2 \neq \sigma_2^2 \neq \dots \neq \sigma_k^2 \end{array} \right\} \text{ Distribusi Normal}$$

tingkat keyakinan $1 - \alpha$

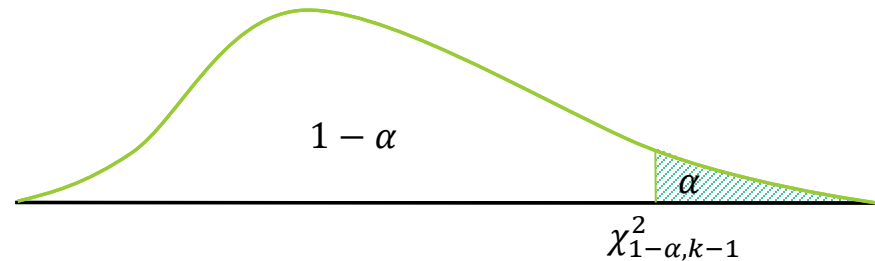
Statistik uji $\frac{Q}{h}$ berdistribusi chi-kuadrat dengan $(k - 1)$ *degrees of freedom*

$$Q = \sum_{i=1}^k (n - 1) \ln \left[\sum_{i=1}^k \frac{(n_i - 1) s_i^2}{N - k} \right] - \sum_{i=1}^k (n - 1) \ln s_i^2$$

H_0 ditolak jika $\frac{Q}{h} > \chi_{1-\alpha, k-1}^2$

$$h = 1 + \frac{1}{3(k - 1)} \sum_{i=1}^k \left(\frac{1}{(n_i - 1)} - \frac{1}{N - k} \right)$$

$$N = \sum_{i=1}^k n_i$$



Uji Hipotesis

- Latihan
 - Lihat kembali data debit puncak tahunan Sungai XYZ.
 - Uji hipotesis yang menyatakan bahwa debit puncak tahunan rerata adalah $650 \text{ m}^3/\text{s}$ dan varians adalah $45.000 \text{ m}^6/\text{s}^2$.
- Contoh uji hipotesis.pdf
- Exercises on hypothesis test.pdf

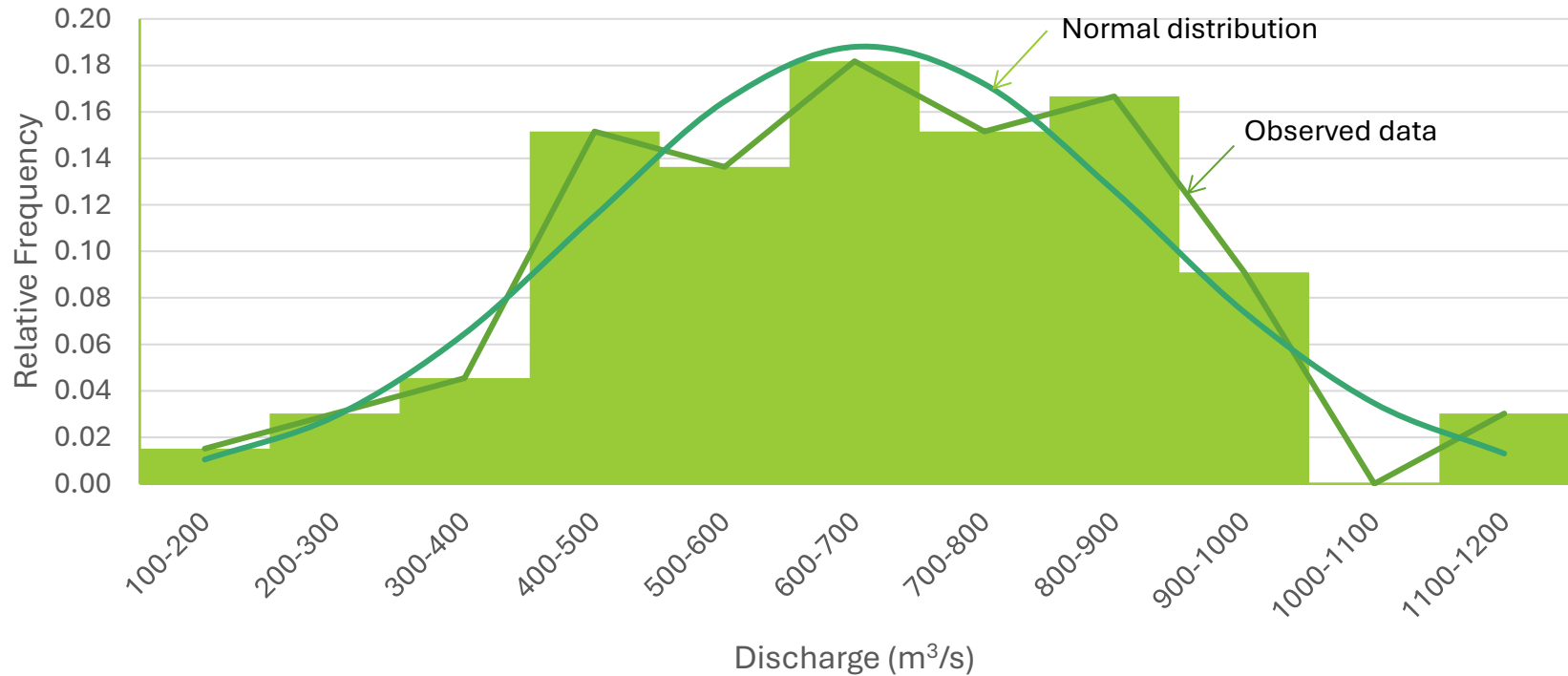
Goodness of Fit Test

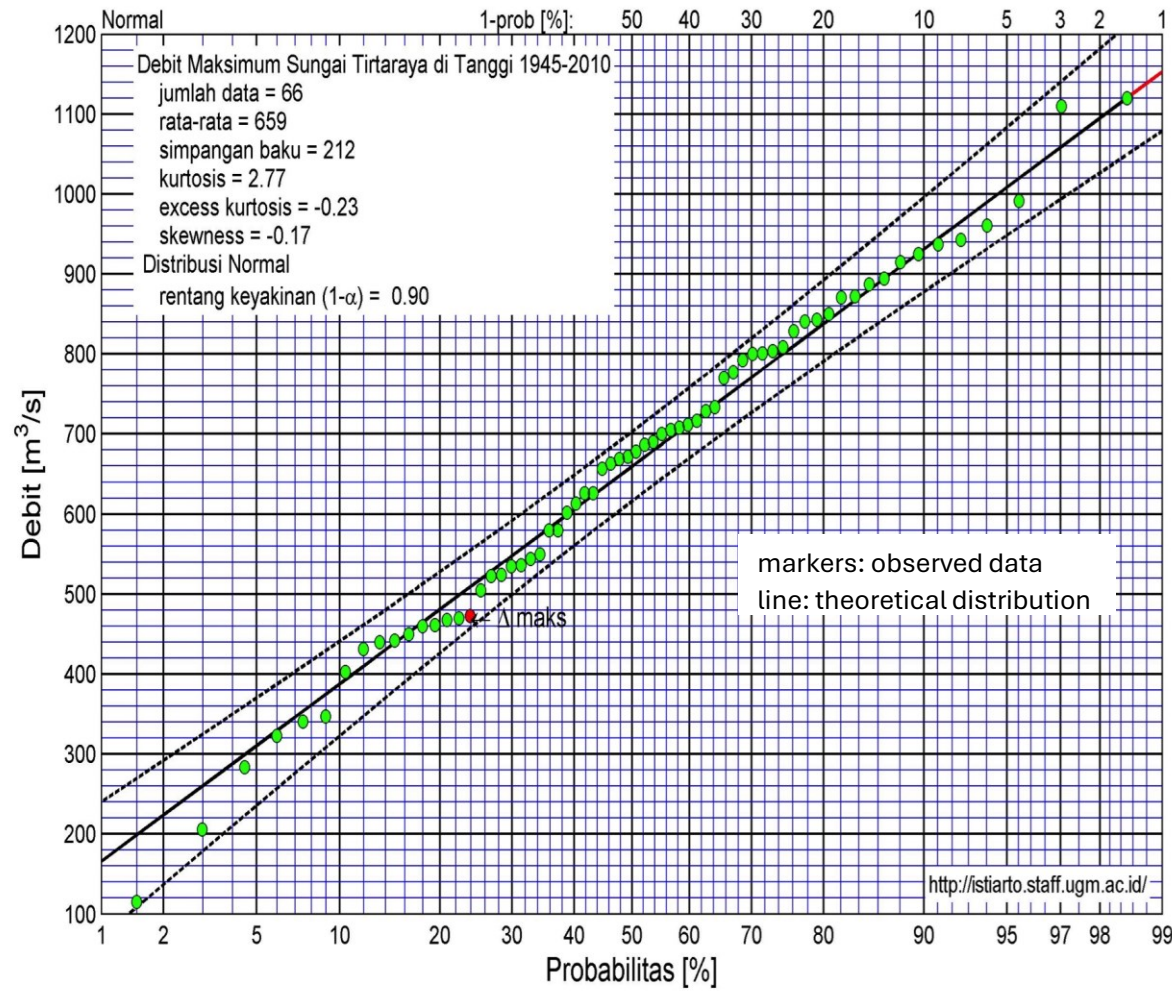
CDF Plot on Probability Paper

Testing The Goodness of Fit of Data to Probability Distributions

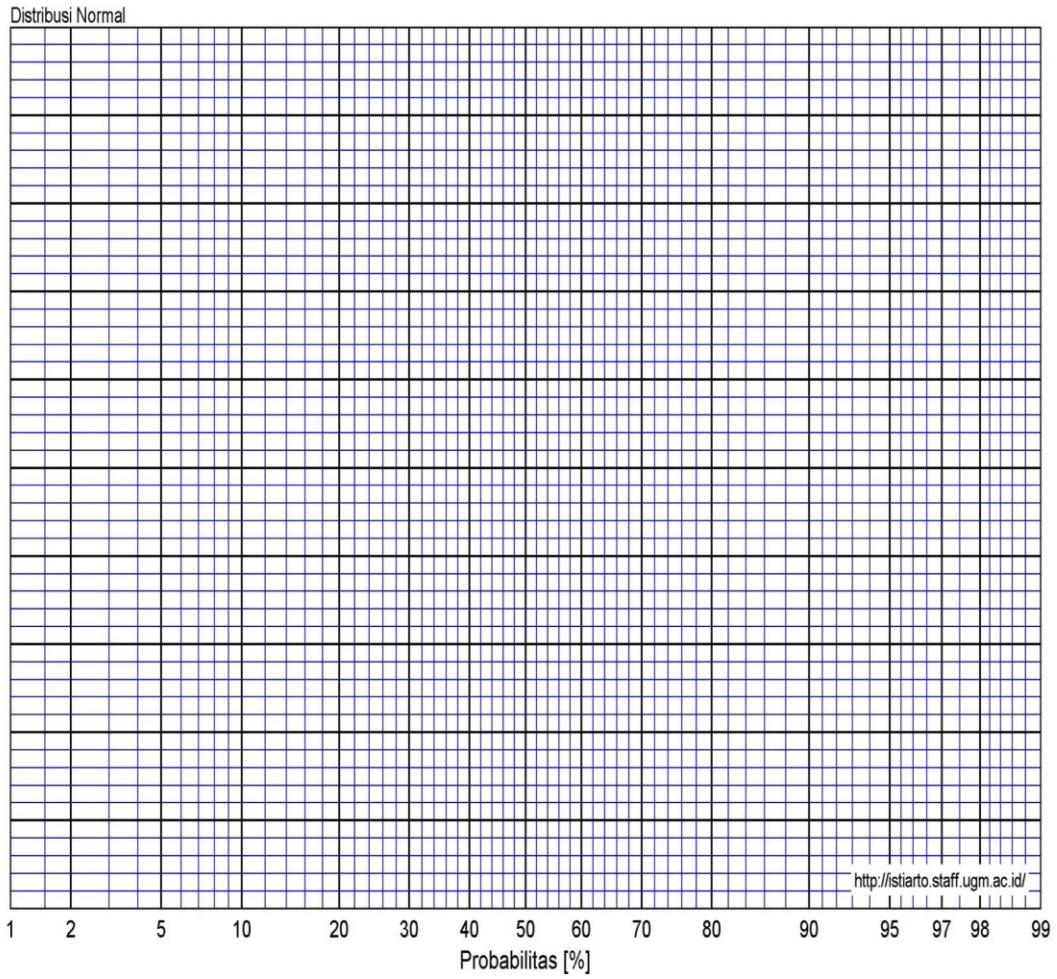
- Graphical (and visual) methods to judge whether or not a particular distribution adequately describes a set of observations
 - plot and compare the observed relative frequency curve with the theoretical relative frequency curve
 - plot the observed data on appropriate probability paper and judge as to whether or not the resulting plot is a straight line
- Statistical tests
 - chi-square goodness of fit test
 - the Kolmogorov-Smirnov test

Annual Peak Discharge of XYZ River





Normal Distribution Paper



Chi-square Goodness of Fit Test

■ Method of test

- Comparison between the actual number of observations and the expected number of observations (expected according to the distribution under test) that fall in the class intervals.
- The expected numbers are calculated by multiplying the expected relative frequency by the total number of observations.
- The test statistic is calculated from the following relationship:

$$\chi_c^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

Chi-square Goodness of Fit Test

- The test statistic is calculated from the following relationship:

$$\chi_c^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

where:

k is the number of class intervals

O_i is the number of observations in the i th class interval

E_i is the expected number of observations in the i th class interval according to the distribution being tested

χ_c^2 has a distribution of chi-square with $(k - p - 1)$ degrees of freedom, where p is the number of parameters estimated from the data

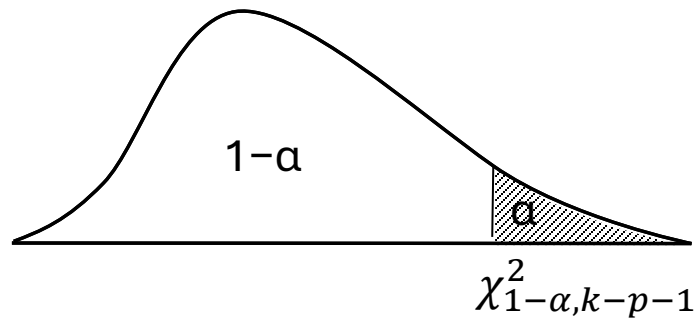
Chi-square Goodness of Fit Test

- The test statistic is calculated from the following relationship

$$\chi_c^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

- The hypothesis that the data are from the specified distribution **is rejected** if

$$\chi_c^2 > \chi_{1-\alpha, k-p-1}^2$$



The Kolmogorov-Smirnov Test

- Steps in the Kolmogorov-Smirnov test
 - Let $P_X(x)$ be the completely specified theoretical cumulative distribution function under the null hypothesis.
 - Let $S_n(x)$ be the sample cumulative density function based on n observations. For any observed x , $S_n(x) = k/n$ where k is the number of observations less than or equal to x .
 - Determine the maximum deviation, D , defined by
$$D = \max |P_X(x) - S_n(x)|$$
 - If, for the chosen significance level, the observed value of D is greater than or equal to the critical tabulated of the Kolmogorov-Smirnov statistic, the hypothesis is rejected. Table of Kolmogorov-Smirnov test statistic is available in many books on statistics.

The Kolmogorov-Smirnov Test

- Notes on the Kolmogorov-Smirnov test
 - The test can be conducted by calculating the quantities $P_X(x)$ and $S_n(x)$ at each observed point or
 - By plotting the data on the probability paper and selecting the greatest deviation on the probability scale of a point from the theoretical line.
 - The data should not be grouped for this test, i.e. plot each point of the data on the probability paper.

Chi-square Goodness of Fit Test and The Kolmogorov-Smirnov Test

■ Exercise

- Do the chi-square goodness of fit test and the Kolmogorov-Smirnov test to the annual peak discharge of XYZ River against normal distribution.

Chi-square Goodness of Fit Test and The Kolmogorov-Smirnov Test

- Notes on both tests when testing hydrologic frequency distributions.
 - Both tests are insensitive in the tails of the distributions.
 - On the other hand, the tails are important in hydrologic frequency distributions.
- To increase sensitivity of chi-square test
 - The expected number of observations in a class shall not be less than 3 (or 5).
 - Define the class interval so that under the hypothesis being tested, the expected number of observations in each class interval is the same.
 - The class intervals will be of unequal width.
 - The interval widths will be a function of the distribution being tested.

Chi-square Goodness of Fit Test and The Kolmogorov-Smirnov Test

- Exercise
 - Redo the chi-square goodness of fit test and the Kolmogorov-Smirnov test to the annual peak discharge of XYZ River against normal distribution.
 - Define the class intervals so that the expected number of observations in each class interval is the same.

Terima kasih