Universitas Gadjah Mada
Fakultas Teknik
Departemen Teknik Sipil dan Lingkungan
Prodi Magister Teknik Sipil

Statistika Teknik

Uji Hipotesis

Uji Hipotesis

- Model Matematika *vs* Pengukuran
  - komparasi garis teoretik (prediksi menurut model) dan data pengukuran
  - jika prediksi model sesuai dengan data pengukuran, maka model diterima
  - jika prediksi model menyimpang dari data pengukuran, maka model ditolak
- Dalam sejumlah kasus, yang terjadi adalah
  - hasil komparasi prediksi model dan data pengukuran tidak cukup jelas untuk menyatakan bahwa model diterima atau ditolak
  - uji hipotesis sebagai alat analisis dalam komparasi tersebut

Prosedur Uji Hipotesis

- Rumuskan hipotesis
- Rumuskan hipotesis alternatif
- Tetapkan statistika uji
- Tetapkan distribusi statistika uji
- Tentukan nilai kritik sebagai batas statistika uji harus ditolak
- Kumpulkan data untuk menyusun statistika uji
- Kontrol posisi statistika uji terhadap nilai kritis

Kemungkinan Kesalahan

Pilihan	Keadaan Nyata	
	Hipotesis Benar	Hipotesis Salah
Menerima	Tidak Salah	Kesalahan Tipe II
Menolak	Kesalahan Tipe I	Tidak Salah

Notasi

H_0 = hipotesis (yang diuji)

H_a = hipotesis alternatif

$1-\alpha$ = tingkat keyakinan (*confidence level*)

Uji Hipotesis Nilai Rerata

H_0 : $\mu = \mu_1$
H_a : $\mu = \mu_2$

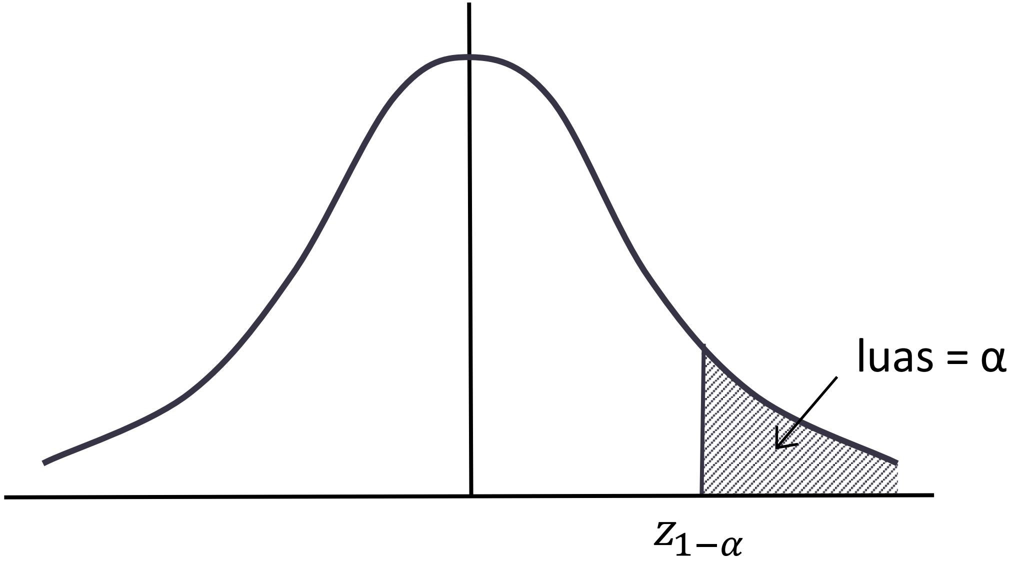
Distribusi Normal
σ_X^2 diketahui

tingkat keyakinan $1-\alpha$

Statistik uji $$Z = \frac{\sqrt{n}}{\sigma_X}\left(\bar{X} - \mu_1\right)$$ berdistribusi normal

Jika $\mu_1 > \mu_2$, H_0 ditolak jika $$\bar{X} < \mu_1 - z_{1-\alpha}\frac{\sigma_X}{\sqrt{n}} \implies Z < -z_{1-\alpha}$$

Jika $\mu_1 < \mu_2$, H_0 ditolak jika $$\bar{X} > \mu_1 + z_{1-\alpha}\frac{\sigma_X}{\sqrt{n}} \implies Z > z_{1-\alpha}$$



$$\text{prob}(Z > z_{1-\alpha}) = \alpha$$

Uji Hipotesis Nilai Rerata

H_0 : $\mu = \mu_1$
H_a : $\mu = \mu_2$

Distribusi Normal
σ_X^2 tidak diketahui

tingkat keyakinan $1-\alpha$

Statistik uji $$T = \frac{\sqrt{n}}{s_X}\left(\bar{X} - \mu_1\right)$$ berdistribusi *t*

Jika $\mu_1 > \mu_2$, H_0 ditolak jika $$\bar{X} < \mu_1 - t_{1-\alpha,n-1}\frac{s_X}{\sqrt{n}} \Rightarrow T < -t_{1-\alpha,n-1}$$

Jika $\mu_1 < \mu_2$, H_0 ditolak jika $$\bar{X} > \mu_1 + t_{1-\alpha,n-1}\frac{s_X}{\sqrt{n}} \Rightarrow T > t_{1-\alpha,n-1}$$

Uji Hipotesis Nilai Rerata

H_0 : μ = μ_0
H_a : μ = μ_0

} Distribusi Normal, $\sigma_X{}^2$ diketahui

tingkat keyakinan 1− α

Statistik uji $$Z = \frac{\sqrt{n}}{\sigma_X}\left(\bar{X} - \mu_0\right)$$ berdistribusi normal

H_0 ditolak jika $$|Z| = \left|\frac{\sqrt{n}}{\sigma_X}\left(\bar{X} - \mu_0\right)\right| > z_{1-\alpha/2}$$

Uji Hipotesis Nilai Rerata

H_0 : $\mu = \mu_0$
H_a : $\mu = \mu_0$

} Distribusi Normal, σ_X^2 tidak diketahui

tingkat keyakinan $1 - \alpha$

Statistik uji $$T = \frac{\sqrt{n}}{s_X}\left(\bar{X} - \mu_0\right)$$ berdistribusi t

H_0 ditolak jika $$|T| = \left|\frac{\sqrt{n}}{s_X}\left(\bar{X} - \mu_0\right)\right| > t_{1-\alpha/2,n-1}$$

Uji Hipotesis Nilai Rerata

- Hasil uji hipotesis adalah
  - menolak H_0, atau
  - tidak menolak H_0
- Artinya
  - H_0: $\mu = \mu_0$
  - Tidak menolak H_0 → "menerima" H_0 berarti bahwa μ tidak berbeda secara signifikan dengan μ_0.
  - Tetapi tidak dikatakan bahwa μ benar-benar sama dengan μ_0 karena kita tidak membuktikan bahwa $\mu = \mu_0$.

Uji hipotesis beda nilai rerata dua buah distribusi normal

H_0 : $\mu_1 - \mu_2 = \delta$

H_a : $\mu_1 - \mu_2 \neq \delta$

} Distribusi Normal
var(X_1) dan var (X_2) diketahui

tingkat keyakinan $1- \alpha$

Statistik uji $$Z = \frac{\bar{X}_1 - \bar{X}_2 - \delta}{\left({\sigma_1}^2/n_1 + {\sigma_2}^2/n_2\right)^{1/2}}$$ berdistribusi normal

H_0 ditolak jika $$|Z| = \left|\frac{\bar{X}_1 - \bar{X}_2 - \delta}{\left({\sigma_1}^2/n_1 + {\sigma_2}^2/n_2\right)^{1/2}}\right| > z_{1-\alpha/2}$$

Uji hipotesis beda nilai rerata dua buah distribusi normal

H_0 : $\mu_1 - \mu_2 = \delta$
H_a : $\mu_1 - \mu_2 \neq \delta$

} Distribusi Normal
var(X_1) dan var (X_2) tidak diketahui

tingkat keyakinan $1-\alpha$

Statistik uji

$$T = \frac{\bar{X}_1 - \bar{X}_2 - \delta}{\left\{\frac{(n_1+n_2)[(n_1-1){s_1}^2 + (n_2-1){s_2}^2]}{[n_1 n_2 (n_1+n_2-2)]}\right\}^{1/2}}$$

berdistribusi t dengan n_1+n_2-2 *degrees of freedom*

H_0 ditolak jika $|T| > t_{1-\alpha/2, n_1+n_2-2}$

Uji Hipotesis Nilai Varians

H_0 : $\sigma^2 = \sigma_0^2$
H_a : $\sigma^2 \neq \sigma_0^2$
} Distribusi Normal

tingkat keyakinan $1-\alpha$

Statistik uji $\chi_c^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{X})}{\sigma_0{}^2}$ berdistribusi chi-kuadrat

H_0 diterima (**tidak** ditolak) jika $\chi^2_{\alpha/2,n-1} < \chi_c^2 < \chi^2_{1-\alpha/2,n-1}$

Uji Hipotesis Nilai Varians

H_0 : $\sigma_1^2 = \sigma_2^2$
H_a : $\sigma_1^2 \neq \sigma_2^2$
} 2 Distribusi Normal

tingkat keyakinan $1-\alpha$

Statistik uji $F_c = \frac{s_1^{\ 2}}{s_2^{\ 2}}$ berdistribusi F dengan
(n_1-1) dan (n_2-1) *degrees of freedom*
$s_1^2 > s_2^2$

H_0 ditolak jika $F_c > F_{1-\alpha,n_1-1,n_2-1}$

Uji Hipotesis Nilai Varians

H_0 : $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

H_a : $\sigma_1^2 \neq \sigma_2^2 \neq \ldots \neq \sigma_k^2$

} Distribusi Normal

tingkat keyakinan $1 - \alpha$

Statistik uji $\frac{Q}{h}$ berdistribusi chi-kuadrat dengan $(k - 1)$ *degrees of freedom*

$$Q = \sum_{i=1}^{k} (n-1) \ln \left[\sum_{i=1}^{k} \frac{(n_i - 1) {s_i}^2}{N - k} \right] - \sum_{i=1}^{k} (n-1) \ln {s_i}^2$$

$$h = 1 + \frac{1}{3(k-1)} \sum_{i=1}^{k} \left(\frac{1}{(n_i - 1)} - \frac{1}{N - k} \right)$$

$$N = \sum_{i=1}^{n} n_i$$

H_0 ditolak jika

$$\frac{Q}{h} > \chi^2_{1-\alpha, k-1}$$

Uji Hipotesis

- Latihan
  - Lihat kembali data debit puncak tahunan Sungai XYZ.
    - Uji hipotesis yang menyatakan bahwa debit puncak tahunan rerata adalah 650 m^3/s dan varians adalah 45.000 m^6/s^2.
- Contoh uji hipotesis.pdf
- *Exercises on hypothesis test*.pdf

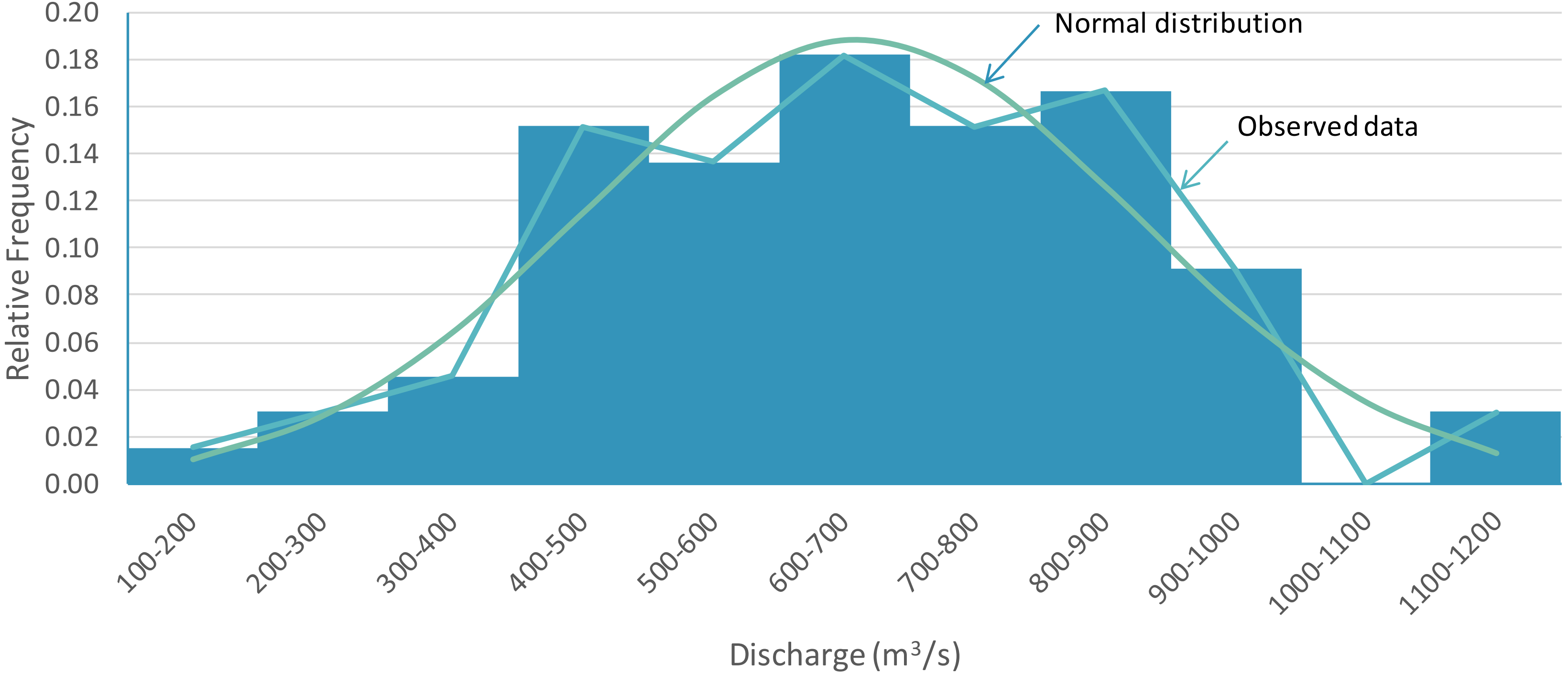
Goodness of Fit Test

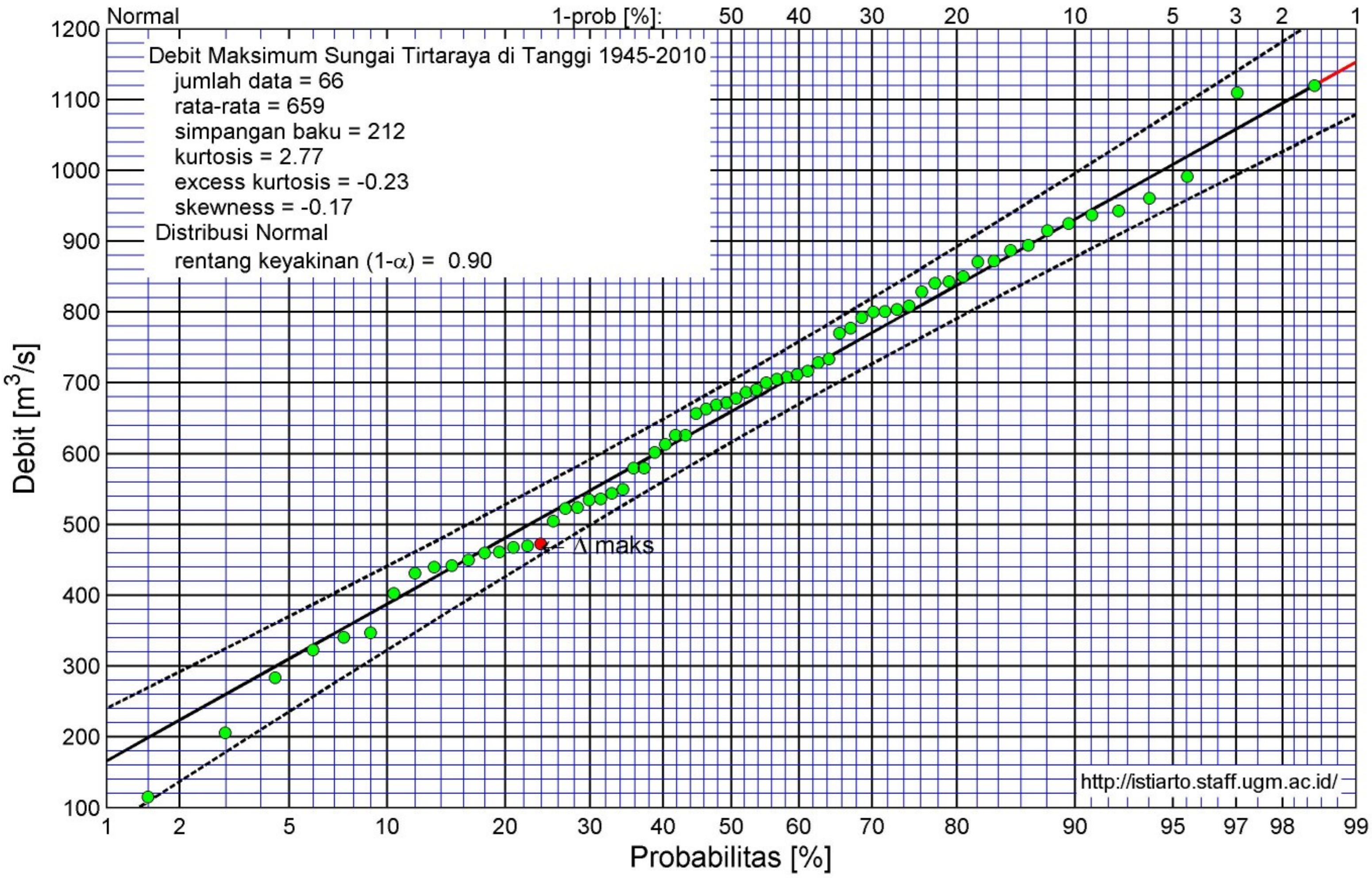
CDF Plot on Probability Paper

Testing The Goodness of Fit of Data to Probability Distributions

- Graphical (and visual) methods to judge whether or not a particular distribution adequately describes a set of observations:
  - plot and compare the observed relative frequency curve with the theoretical relative frequency curve
  - plot the observed data on appropriate probability paper and judge as to whether or not the resulting plot is a straight line
- Statistical tests:
  - chi-square goodness of fit test
  - the Kolmogorov-Smirnov test

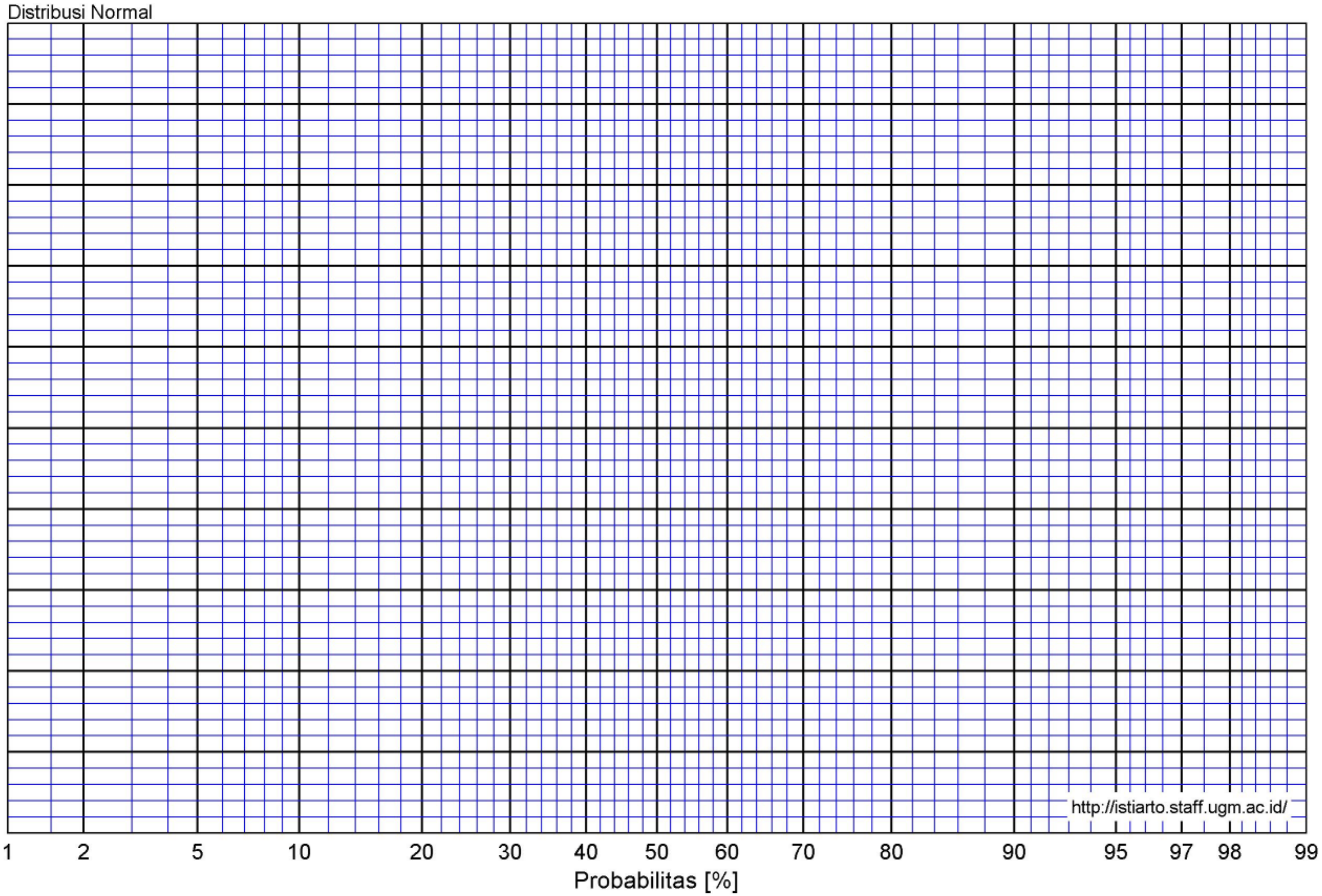
Annual Peak Discharge of XYZ River





Normal
1-prob [%]:
50 40 30 20 10 5 3 2 1
Debit Maksimum Sungai Tirtaraya di Tanggi 1945-2010
jumlah data = 66
rata-rata = 659
simpangan baku = 212
kurtosis = 2.77
excess kurtosis = -0.23
skewness = -0.17
Distribusi Normal
rentang keyakinan (1-α) = 0.90
Δ maks
Debit [m3/s]
1200 1100 1000 900 800 700 600 500 400 300 200 100
1 2 5 10 20 30 40 50 60 70 80 90 95 97 98 99
Probabilitas [%]
http://istiarto.staff.ugm.ac.id/

Normal Distribution Paper



Chi-square Goodness of Fit Test

- Method of test
  - Comparison between the actual number of observations and the expected number of observations (expected according to the distribution under test) that fall in the class intervals.
  - The expected numbers are calculated by multiplying the expected relative frequency by the total number of observations.
  - The test statistic is calculated from the following relationship:

$$\chi_c^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

Chi-square Goodness of Fit Test

- The test statistic is calculated from the following relationship:

$$\chi_c^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where:

k is the number of class intervals

O_i is the number of observations in the *i*th class interval

E_i is the expected number of observations in the *i*th class interval according to the distribution being tested

χ_c^2 has a distribution of chi-square with ($k - p - 1$) degrees of freedom, where *p* is the number of parameters estimated from the data

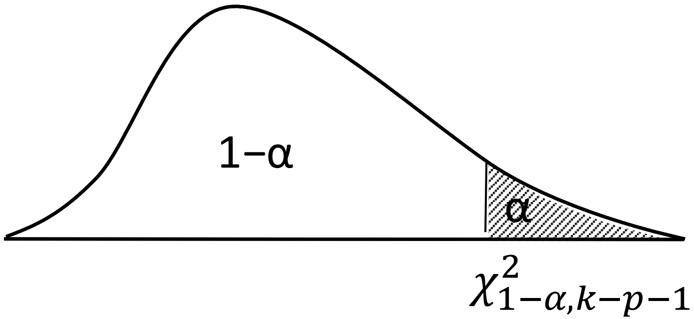
Chi-square Goodness of Fit Test

- The test statistic is calculated from the following relationship:

$$\chi_c^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

- The hypothesis that the data are from the specified distribution **is rejected** if:

$$\chi_c^2 > \chi^2_{1-\alpha,k-p-1}$$



The Kolmogorov-Smirnov Test

- Steps in the Kolmogorov-Smirnov test:
  - Let $P_X(x)$ be the completely specified theoretical cumulative distribution function under the null hypothesis.
  - Let $S_n(x)$ be the sample comulative density function based on n observations. For any observed x, $S_n(x) = k/n$ where k is the number of observations less than or equal to x.
  - Determine the maximum deviation, D, defined by:
    $D = \max |P_X(x) - S_n(x)|$
  - If, for the chosen significance level, the observed value of D is greater than or equal to the critical tabulated of the Kolmogorov-Smirnov statistic, the hypothesis is rejected. Table of Kolmogorov-Smirnov test statistic is available in many books on statistics.

The Kolmogorov-Smirnov Test

- Notes on the Kolmogorov-Smirnov test:
  - The test can be conducted by calculating the quantities $P_X(x)$ and $S_n(x)$ at each observed point or
  - By plotting the data on the probability paper and and selecting the greatest deviation on the probability scale of a point from the theoretical line.
    - The data should not be grouped for this test, i.e. plot each point of the data on the probability paper.

Chi-square Goodness of Fit Test and The Kolmogorov-Smirnov Test

- Exercise
  - Do the chi-square goodness of fit test and the Kolmogorov-Smirnov test to the annual peak discharge of XYZ River against normal distribution.

Chi-square Goodness of Fit Test and The Kolmogorov-Smirnov Test

- Notes on both tests when testing hydrologic frequency distributions.
  - Both tests are insensitive in the tails of the distributions.
  - On the other hand, the tails are important in hydrologic frequency distributions.
- To increase sensitivity of chi-square test
  - The expected number of observations in a class shall not be less than 3 (or 5).
  - Define the class interval so that under the hypothesis being tested, the expected number of observations in each class interval is the same.
    - The class intervals will be of unequal width.
    - The interval widths will be a function of the distribution being tested.

Chi-square Goodness of Fit Test and The Kolmogorov-Smirnov Test

- Exercise
  - Redo the chi-square goodness of fit test and the Kolmogorov-Smirnov test to the annual peak discharge of XYZ River against normal distribution.
    - Define the class intervals so that the expected number of observations in each class interval is the same.

Terima kasih