STATISTICS AND PROBABILITY

First Semester 2024-2025

Midterm Exam

Wednesday, October 2, 2024

A closed-book, two-hour exam

Part A (20%)

Do either one of the following two problems.

Problem A.1 (PI a1, a2, a3; 20%)

Concrete supply for a construction project comes from three batching plants, namely plant A, B, and C. The probabilities that the concrete comes from those plants are, 30%, 45%, and 25%, respectively. The quality engineer of the construction project observes that concrete supply from each plant has a risk of being late. The probability of late supply from A, B, and C plants are 12%, 10%, and 7%, respectively.

a. Find the probability of late concrete supply to the project.
b. Suppose that a late supply has occurred. Find the probability that the supply came from batching plant B.

Problem A.2 (PI a1, a2, a3; 20%)

The following data are the yield strength, f_y, in mega pascal (MPa) of one of the reinforcing steel columns of building columns.

472	431	455	431	425
415	430	427	437	421
429	432	431	425	419

a. Find the mean and the standard deviation of the sample.
b. Find the lower quartile and upper quartile of the sample.
c. The design yield of the reinforcing steel is defined at 420 MPa. Could we conclude that the steel column meets this design criterion? Give your argument to support your conclusion.

Part B (80%)

Do two out of four of the following problems.

Problem B.1 (PI a1, a2, a3; 40%)

The probability density function of the annual series of maximum daily rainfall at a weather station, H in millimeters, is expressed as follow

$$p_H(h) = \begin{cases} 0 & \text{if } h < 0 \\ \dfrac{1}{75} & \text{if } 0 < h < 50 \\ \dfrac{1}{3750}(100 - h) & \text{if } 50 < h < 100 \\ 0 & \text{if } h > 100 \end{cases}$$

a. Draw the probability density function curve.
b. Find and draw the cumulative distribution function curve.
c. Find prob(40 mm < H < 60 mm).
d. Find the median of the maximum daily rainfall.

Problem B.2 (PI a1, a2, a3; 40%)

Table below shows the frequency of daily air temperature in centigrade and relative air humidity in percent at a weather station.

		Air temperature (°C)					
		22—24	24—26	26—28	28—30	30—32	32—34
Air humidity (%)	0—20	2	4	6	2	2	1
	20—40	4	8	12	30	6	9
	40—60	5	15	30	60	30	20
	60—80	3	7	9	25	17	11
	80—100	1	0	2	12	8	3

a. Find the joint probability density function of the two random variables.
b. Find the marginal pdf and cdf of the air temperature.
c. Find the marginal pdf and cdf of the air humidity.
d. Find the probability of the air temperature be in the range of 28°C to 30°C.
e. Find the probability of the air temperature be in the range of 28°C to 30°C when the air relative humidity is in the range of 60% to 80%.

Problem B.3 (PI a1, a2, a3; 40%)

A flood early warning in a river relies on measured water level at a cross section. It is known that the instrument has an error probability of 1/1000.

a. Find the probability of no more than 3 errors in 1000 measurements. Use Poisson distribution.
b. Redo the above problem by applying binomial distribution.
c. Consider problems a and b. Which approach is more suitable to predict the probability of the event?

Problem B.4 (PI a1, a2, a3; 40%)

Laboratory test data show that the compressive strength, f'_c, of concrete samples is normally distributed whose mean and standard deviation are, respectively, 27 MPa 2.5 MPa.

a. Find the probability that a concrete sample has a compressive strength between 25 MPa and 28 MPa, $\text{prob}\left(25\text{ MPa} < f'_c < 28\text{ MPa}\right)$.
b. SNI 2847:2019 dictates that the minimum compressive strength of concrete as building material is 21 MPa. Does the sample meet the standard?

Notes

Sample mean

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Standard deviation of sample

$$s_X = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{X})^2}{n-1}}$$

Binomial distribution

$$f_X(x;n,p) = \binom{n}{x}\ p^x(1-p)^{n-x}, \quad x = 0,\ 1,\ 2,\ \dots,\ n$$

$$F_X(x;n,p) = \sum_{i=0}^{x} f_X(i;n,p) =$$

$$\sum_{i=0}^{x}\binom{n}{i}\ p^i(1-p)^{n-i}\,,\ x = 0,1,2,\dots,n$$

Poisson distribution

$$f_X(x;\lambda) = \frac{\lambda^x\ e^{-\lambda}}{x!}\,, x = 0,1,2,\dots.\ \text{ and } \lambda = np > 0$$

$$F_X(x;\lambda) = \sum_{i=0}^{x}\frac{\lambda^i\ e^{-\lambda}}{i!}$$

Total probability

$$\text{prob}(A) = \sum_{i=1}^{n}\text{prob}(B_i)\ \text{prob}(A|B_i)$$

Bayes theorem

$$\text{prob}(B_j|A) = \frac{\text{prob}(B_j)\ \text{prob}(A|B_j)}{\sum_{i=1}^{n}\text{prob}(B_i)\ \text{prob}(A|B_i)}$$

CDF *vs* PDF

$$P_H(h) = \int_{t_1}^{t_2} p_H(t)\ dt$$

Standard normal distribution

$$Z_x = \frac{X-\mu}{\sigma}$$

$$\text{prob}(Z > z) = 1 - \text{prob}(Z < z)$$

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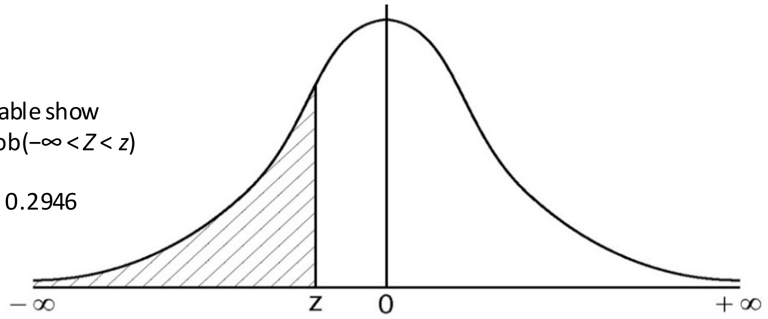
Area under the pdf of standard normal distribution from −∞ to *z*

numbers in the table show
shaded area = prob(−∞ < *Z* < *z*)
example
prob(*Z* < −0.54) = 0.2946



z	0	1	2	3	4	5	6	7	8	9
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000